Image Analysis for Volumetric Industrial Inspection and Interaction

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DTU Informatics

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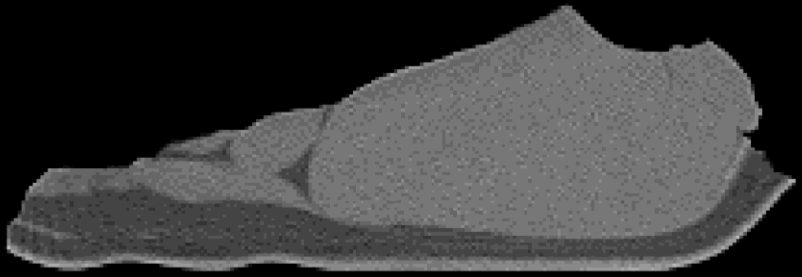
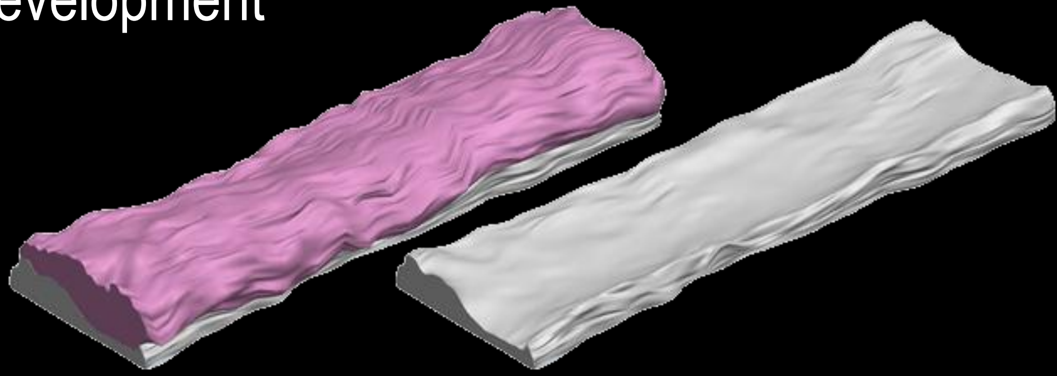
- (CT) reconstruction from few projections
- Interpolation of low resolution images
- The Virtual Knife - interacting with 3D volumes with fast visual feedback

Motivation - Industrial application of CT

- Fast, inexpensive scanning using a minimum of projections
- Scanning pig backs
    - Quality assesment (lean meat, fat %)
    - Robot control – rind trimming
    - Visualization – product development

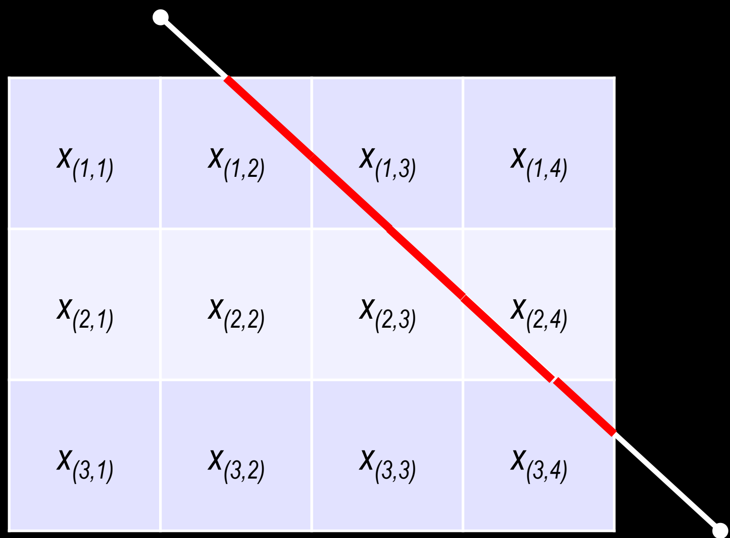




Ray Attenuation Model

- Discretise line integral of attenuation coefficients
- The measured attenuation b_j is a weighted sum of pixels traversed by the ray
- Pixel weights - length of ray's path through it

$$b_j = a_1 x_{(1,2)} + a_2 x_{(1,3)} + a_3 x_{(2,3)} + a_4 x_{(2,4)} + a_5 x_{(3,4)}$$

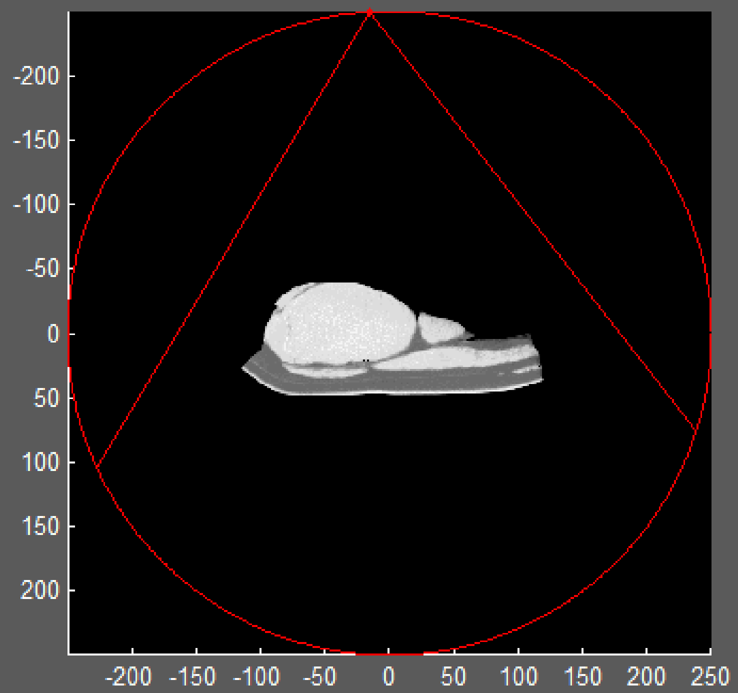


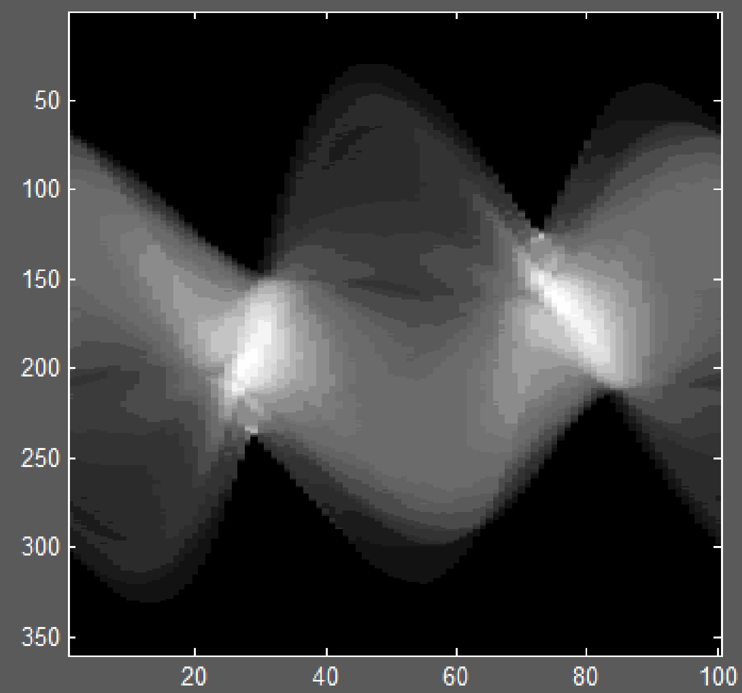
- Resulting linear system

$$\mathbf{b} = \mathbf{A}\mathbf{x}$$

Fan beam - Image Acquisition

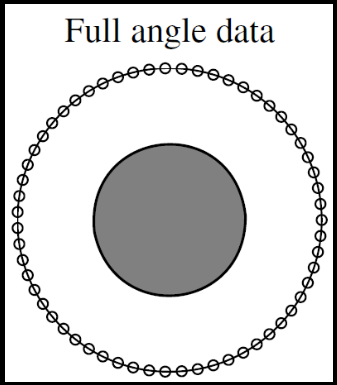
- 100 fan-beam projections of 360 rays each
- Measured on arc-shaped detector array opposite source

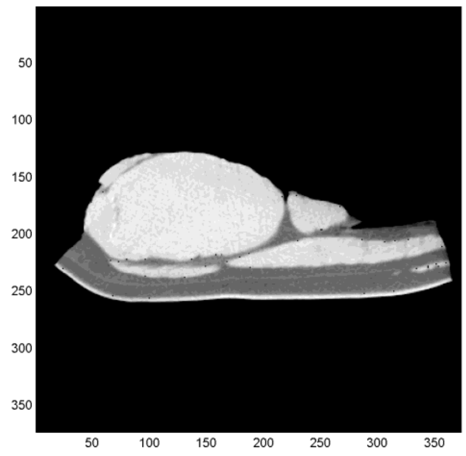


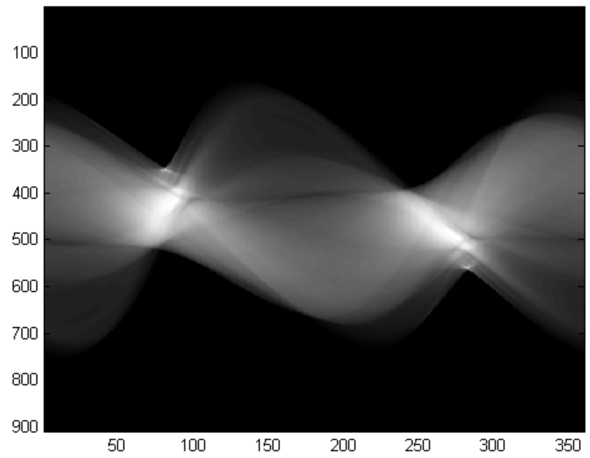


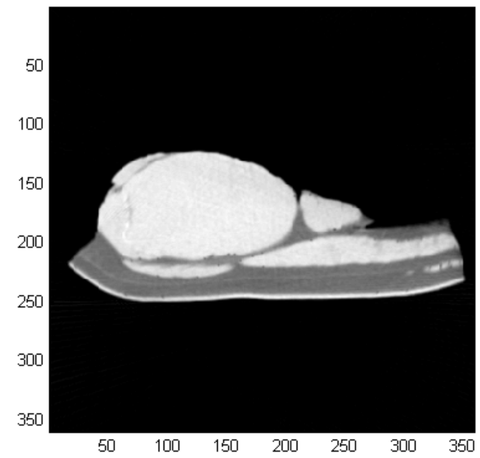
Filtered Back Projection

- Does well with *full angle data*
  - 360x360 image from
    360 source positions with 909 rays each









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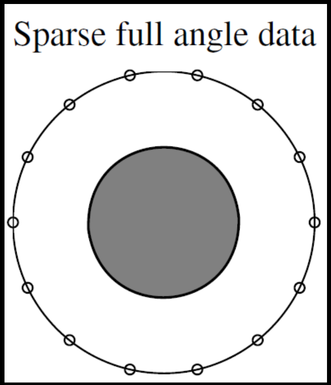
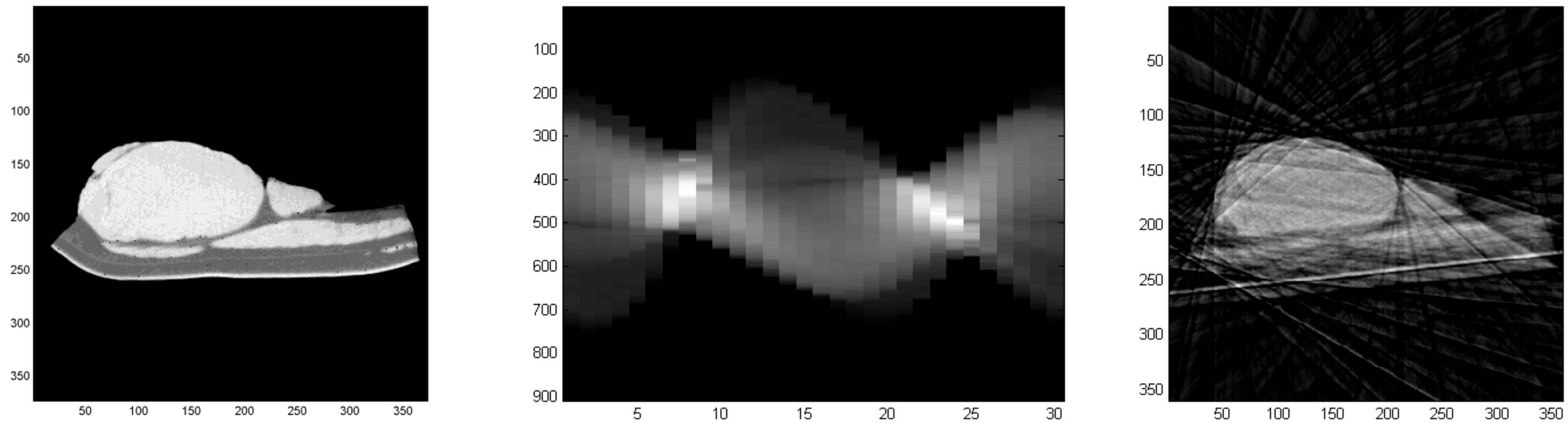
Filtered Back Projection

- Not so good with *sparse data*
  - fewer source positions
  - 30 source positions, 909 rays each





Reformulate – Probabilistic Approach

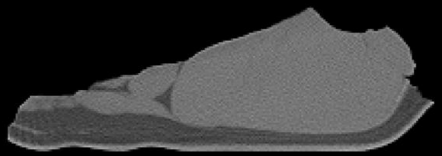
- Bayes' theorem

$$p(\mathbf{x}|\mathbf{b}) \propto p(\mathbf{x})\ p(\mathbf{b}|\mathbf{x})$$

- The loglikelihood of the data

$$\log p(\mathbf{b}\,|\,\mathbf{x}) = \left\|\mathbf{b}-\mathbf{A}\mathbf{x}\right\|^2 / 2\sigma^2 + c$$

- Pig CTs are piece-wise constant – prior should
  - penalise intensity roughness
  - allow for *jumps* on edges



Prior Formulation

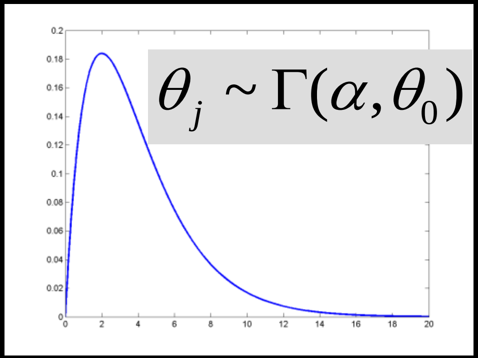
- Penalise large gradients - finite difference operators: $\mathbf{L}_1\mathbf{x}$, $\mathbf{L}_2\mathbf{x}$

$$\log p(\mathbf{x}) = -\tfrac{1}{2}\left(\left\|\mathbf{L}_1\mathbf{x}\right\|^2 + \left\|\mathbf{L}_2\mathbf{x}\right\|^2\right) + c = -\tfrac{1}{2}\mathbf{x}^T\left(\mathbf{L}_1^T\mathbf{L}_1 + \mathbf{L}_2^T\mathbf{L}_2\right)\mathbf{x} + c$$

- But only at a sparse set of edge points

$$\log p(\mathbf{x}|\boldsymbol{\theta})p(\boldsymbol{\theta}) = -\tfrac{1}{2}\mathbf{x}^T\left(\mathbf{L}_1^T\mathbf{D}^{-1}\mathbf{L}_1 + \mathbf{L}_2^T\mathbf{D}^{-1}\mathbf{L}_2\right)\mathbf{x} + \log p(\boldsymbol{\theta}) + c$$

$$\mathbf{D} = \mathrm{diag}(\theta_1, \cdots, \theta_N)$$



Reconstruction

- Choose (x,θ) to maximise the posterior probability
- Conjugated Gradient Least Squares
- Two-step algorithm.
  - Fix θ, minimise wrt. X
  - Fix x, minimise wrt. Θ

- Works also for cone beam CT.

... Now some images!

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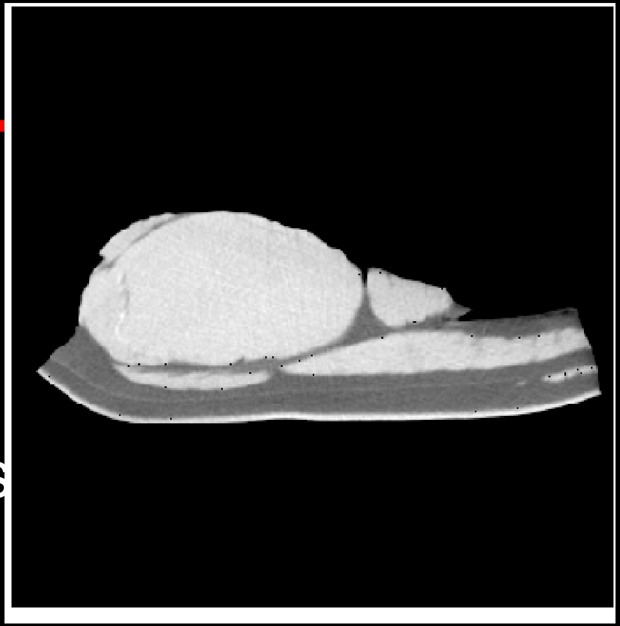
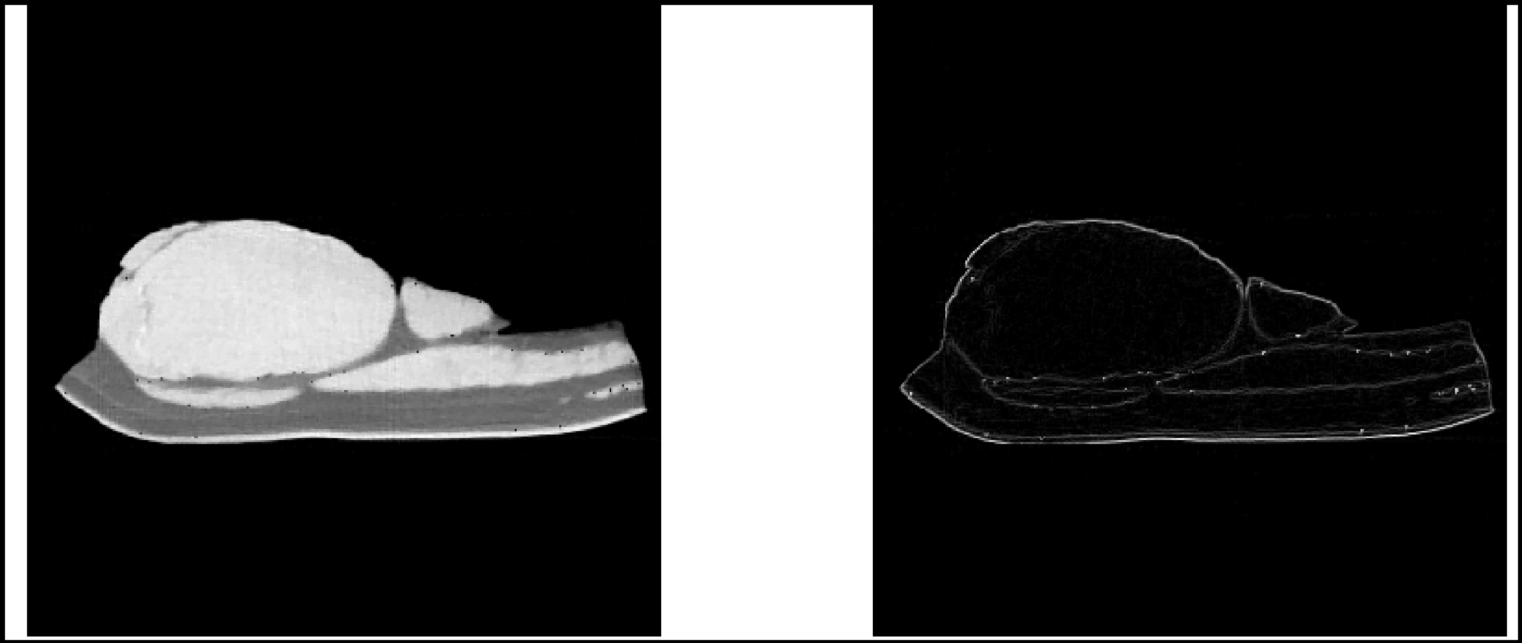
Results

- Only few two-step-sweeps needed...

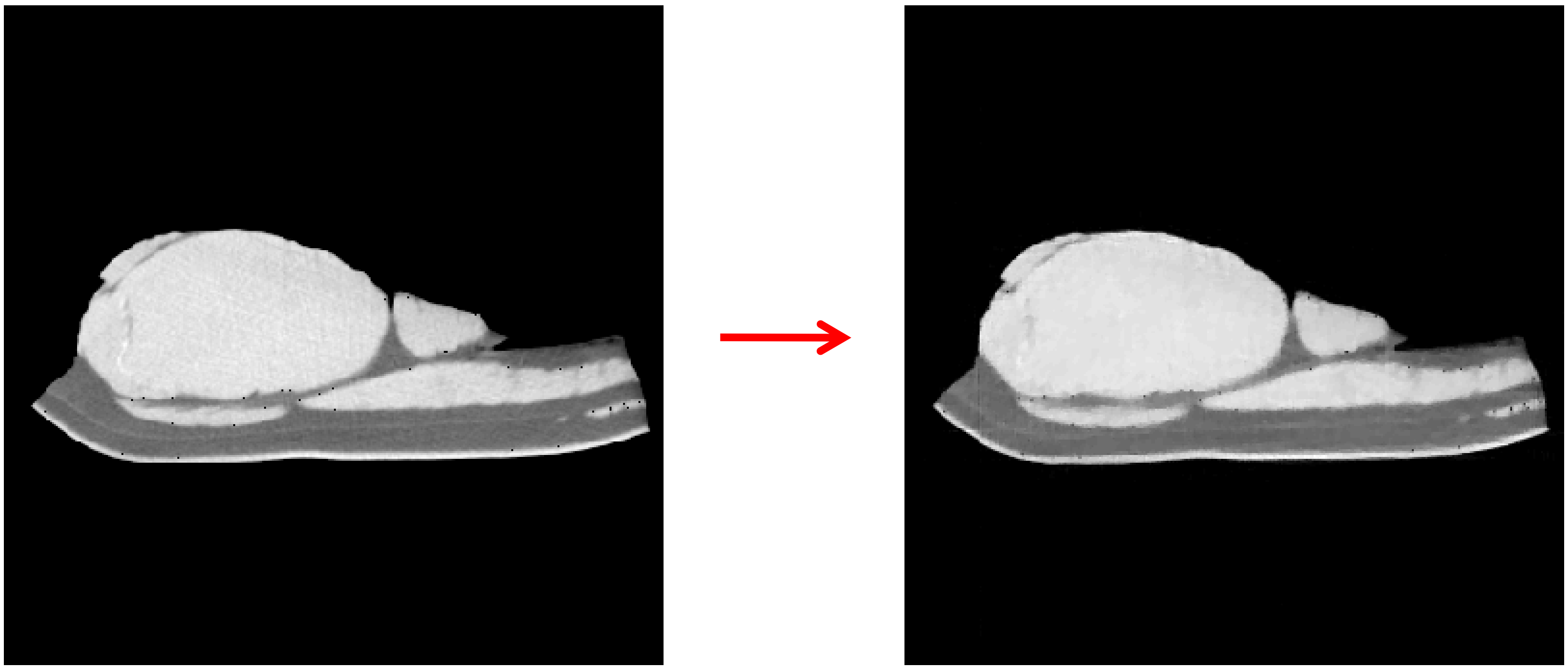
- 360x360 image from 100 fans of 540 rays
- The prior acts as wanted





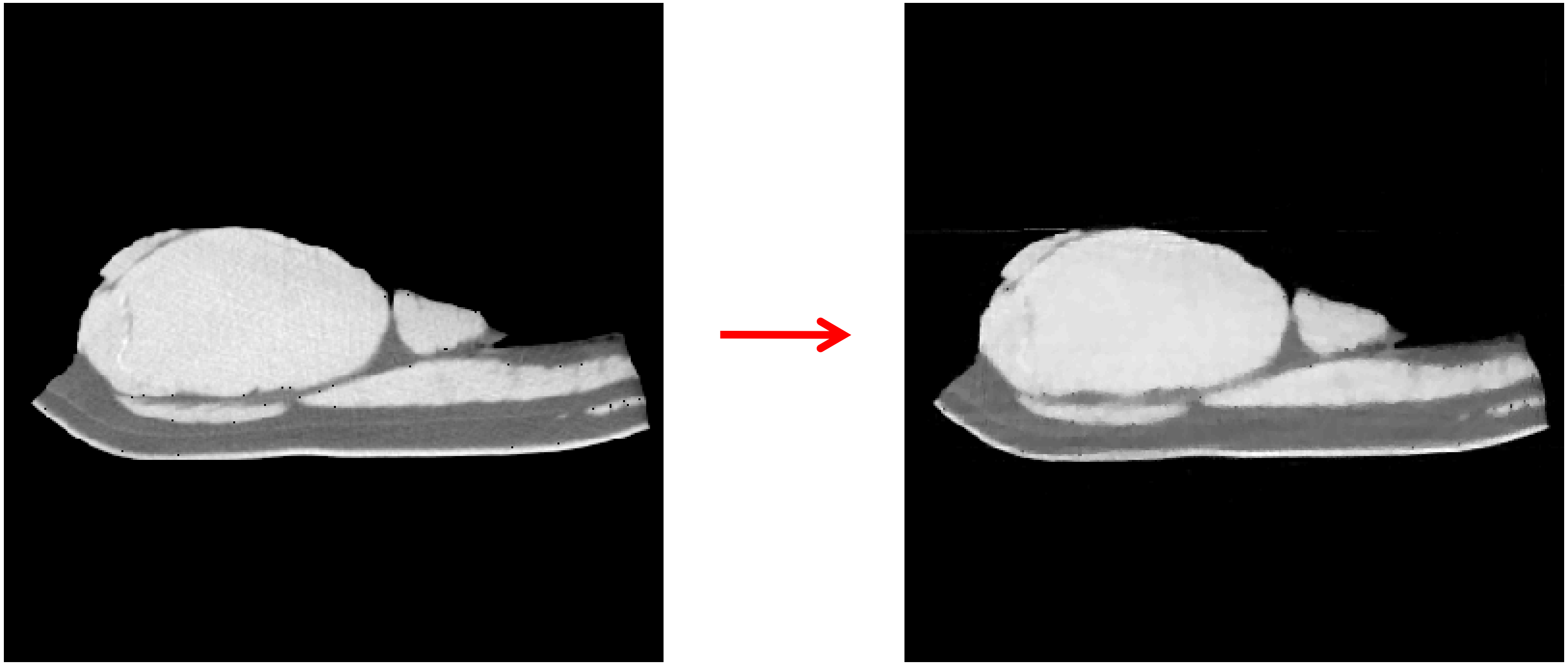
Results

- 50 fans of 540 rays



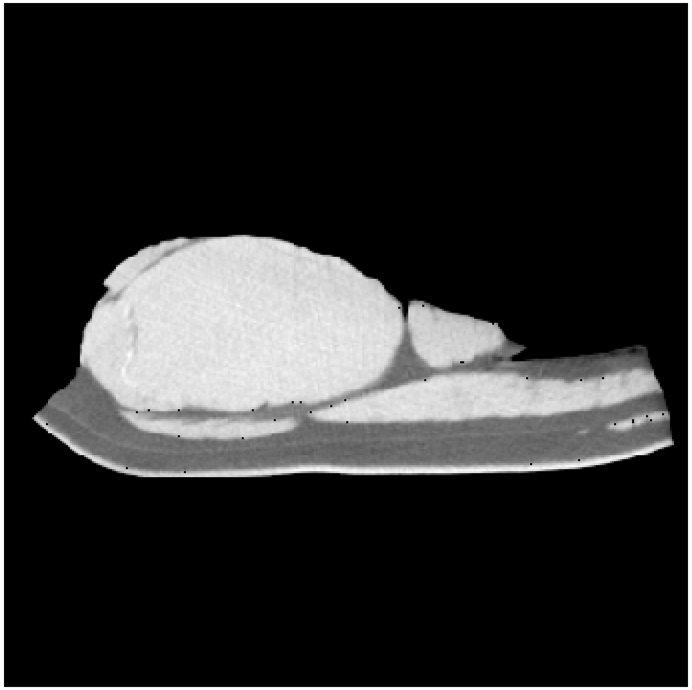
Results

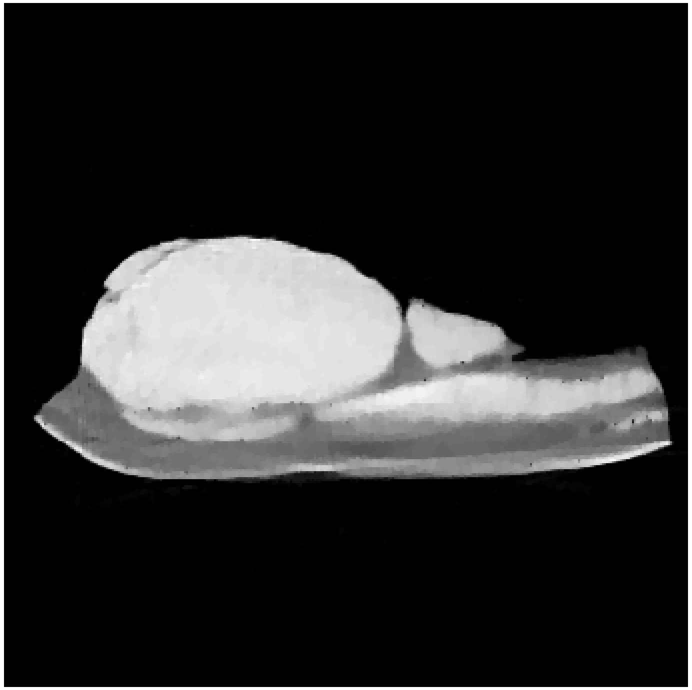
- 40 fans of 540 rays



Results

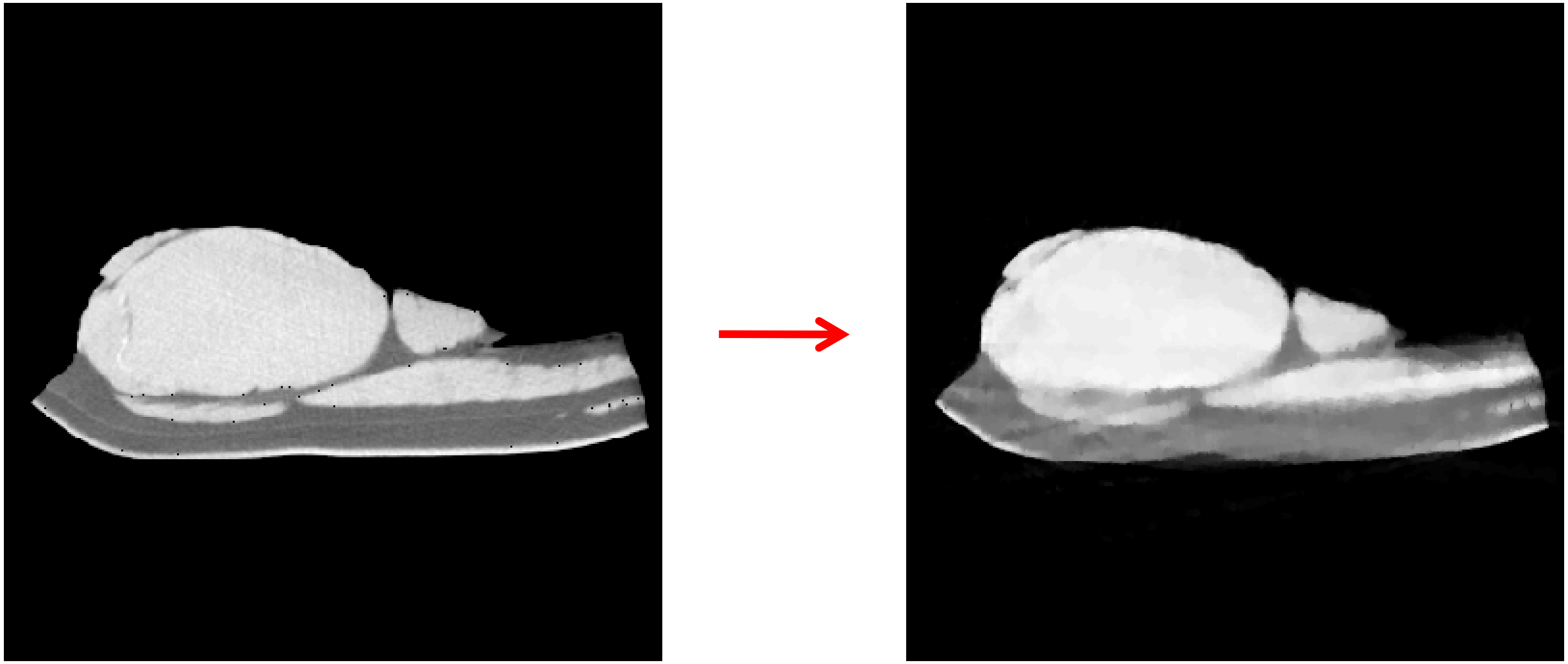
- 30 fans of 540 rays





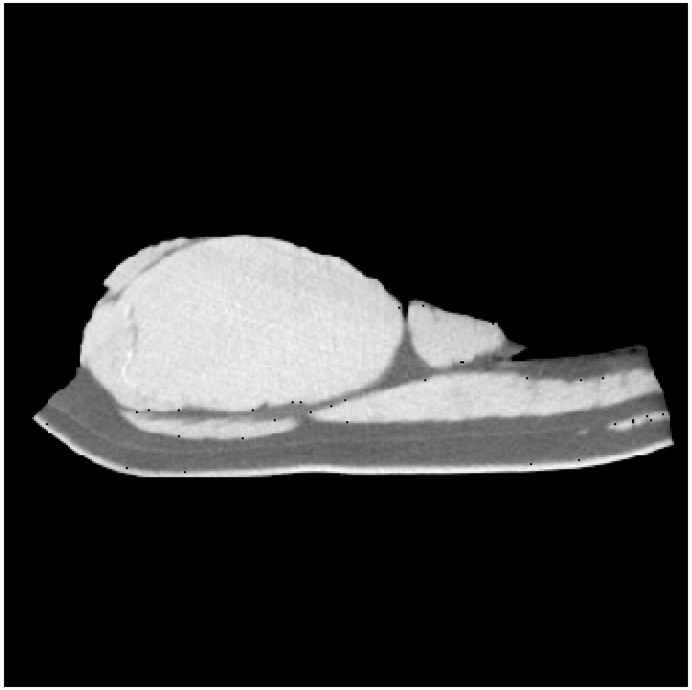
Results

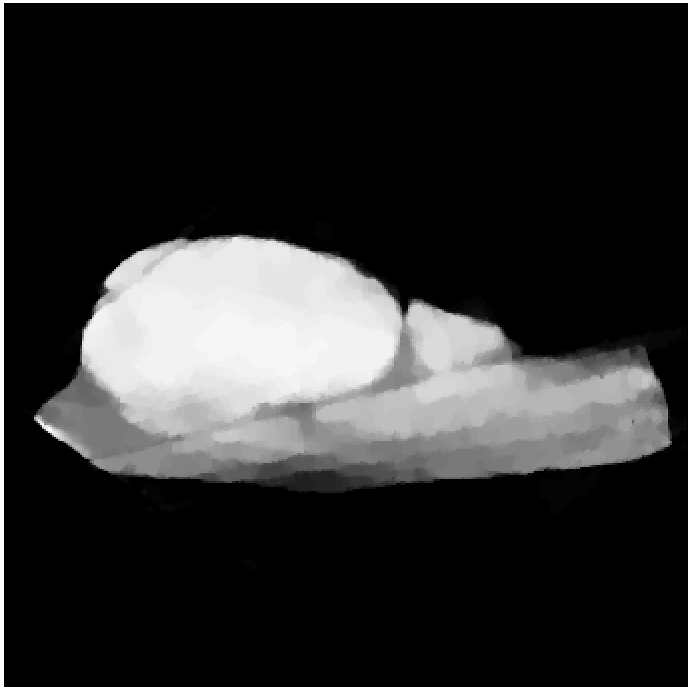
- 20 fans of 540 rays (pushing it)



Results

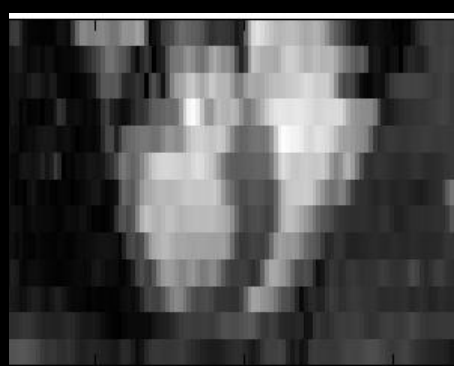
- 10 fans of 540 rays (***really*** pushing it)



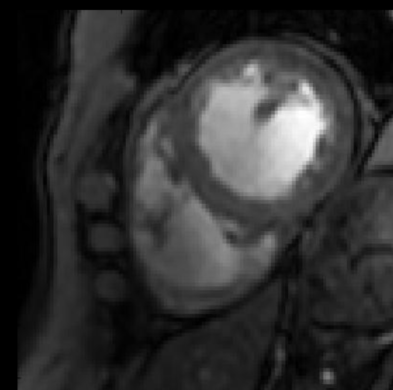


Motivation - Interpolation

- In-plane resolution higher than through plane
- Also to compensate for motion



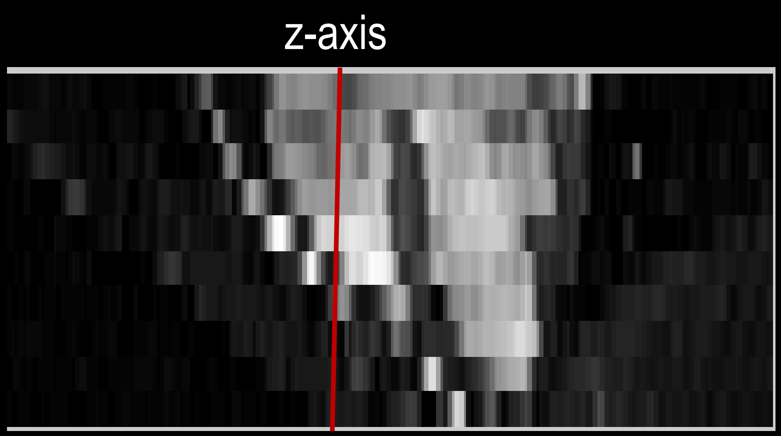
Through-plane: 8mm slice thickness + 1mm gap



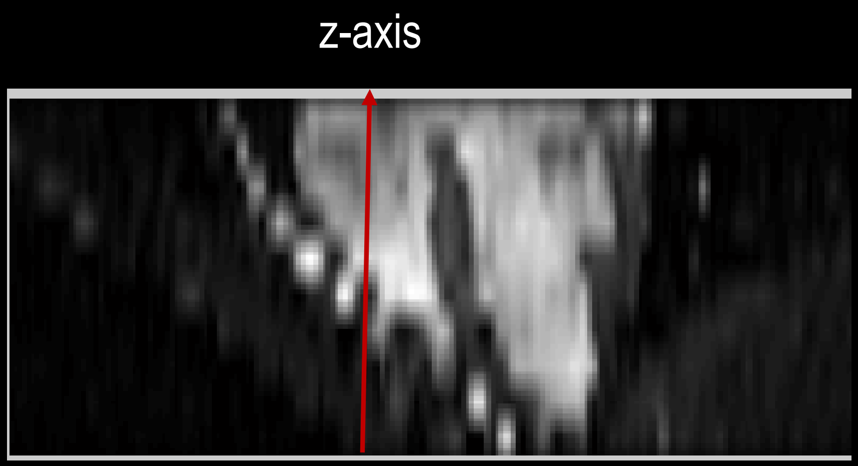
In-plane: 1mm

Interpolation

- Linear interpolation: Weighted average of neighboring voxels



Original



Linear

Improved correspondence-based interpolation

- Adjacent slices: I_A, I_B
- Register I_A to I_B and vice versa, obtain displacements w_{AB}, w_{BA}
- z-position of desired interpolated value, *I* gives the distance ratio,

  $\alpha = (z_I - z_B)/(z_A - z_B)$
- Determine intensities at z_I between I_A and I_B:

$$\mathrm{I} = (1-\alpha)\mathrm{I_A}(\varphi_{\mathrm{BA}}(\mathrm{X}, \alpha \mathrm{w_{BA}})) + \alpha \mathrm{I_B}(\varphi_{\mathrm{AB}}(\mathrm{X}, (1-\alpha)\mathrm{w_{AB}}))$$

Penney et al . Registration-based Interpolation, IEEE-TMI 2004

Frakes et al. A New method for Registration-based Interpolation, IEEE-TMI 2008

Olafdottir et al. Improving Image Registration using Correspondence Interpolation, ISBI 2011

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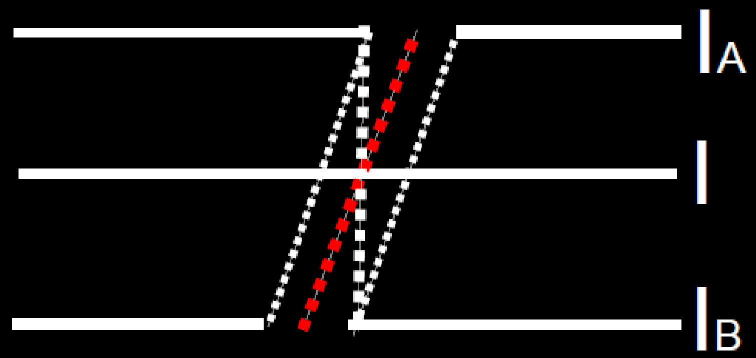
Formulation with respect to Linear Interpolation

Improved Correspondence-based interpolation

$$\mathbf{I} = (1-\alpha)\mathbf{I_A}(\varphi(\mathbf{X}, \alpha\mathbf{w_{BA}})) + \alpha\mathbf{I_B}(\varphi(\mathbf{X}, (\mathbf{1}-\alpha)\mathbf{w_{AB}}))$$

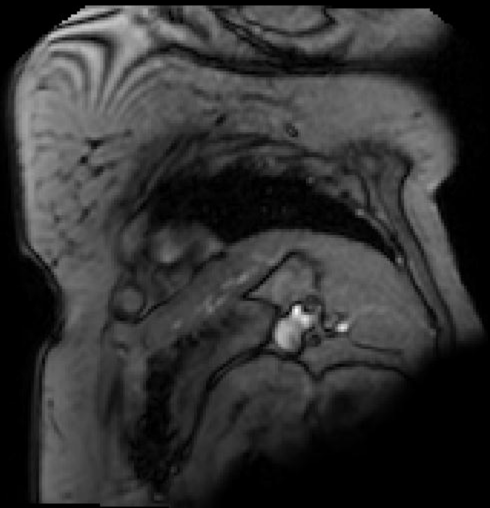
Linear Interpolation

$$\mathbf{I} = (1-\alpha)\mathbf{I_A}(\mathbf{X}) + \alpha\mathbf{I_B}(\mathbf{X})$$

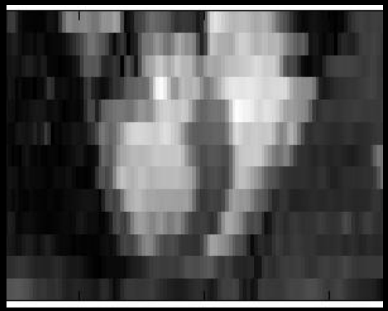


Approximation: Assuming smoothness and dense correspondence field

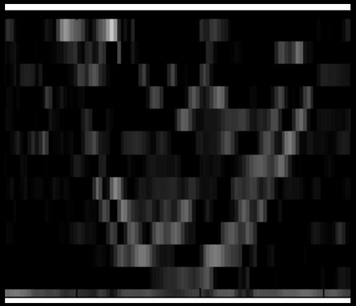
Cardiac example



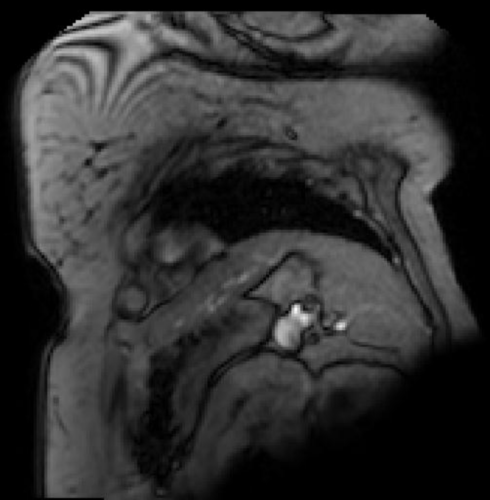
Original



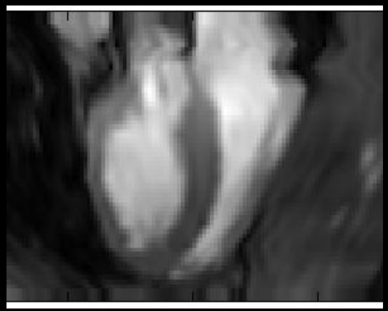
Original



z-gradient with
linear interpolation



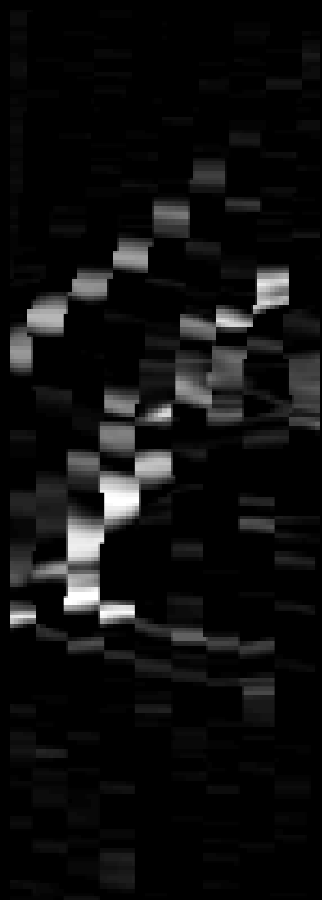
Correspondence-based interp



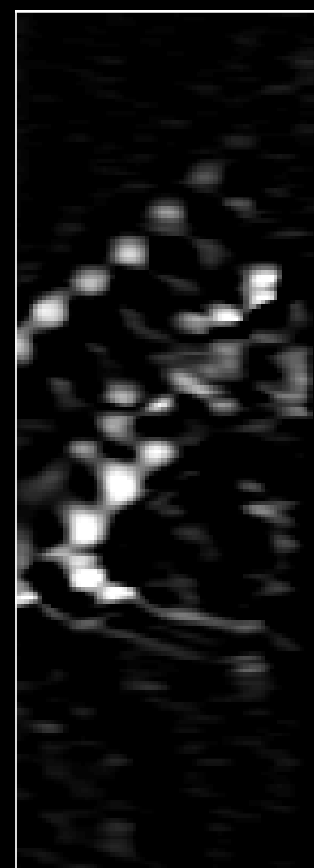
Correspondence-based

z-gradient with
correspondence-based

Gradients of deformed targets, $\frac{\partial \mathrm{T}}{\partial \varphi}$ cardiac MRI



Linear



Spline

Correspondence

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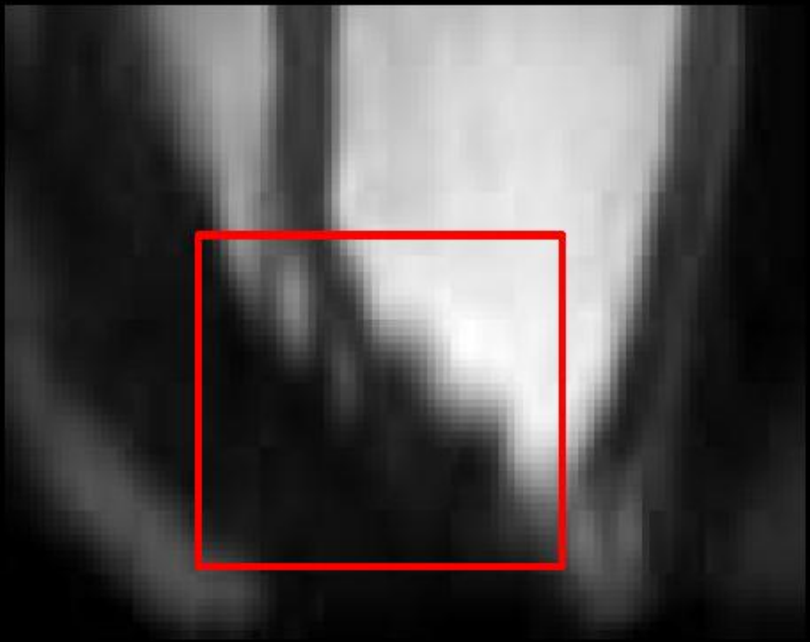
Atlas Building



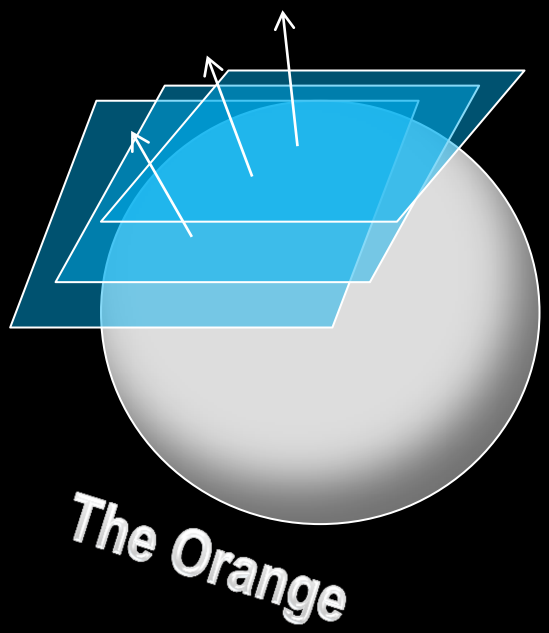
Linear

Correspondence

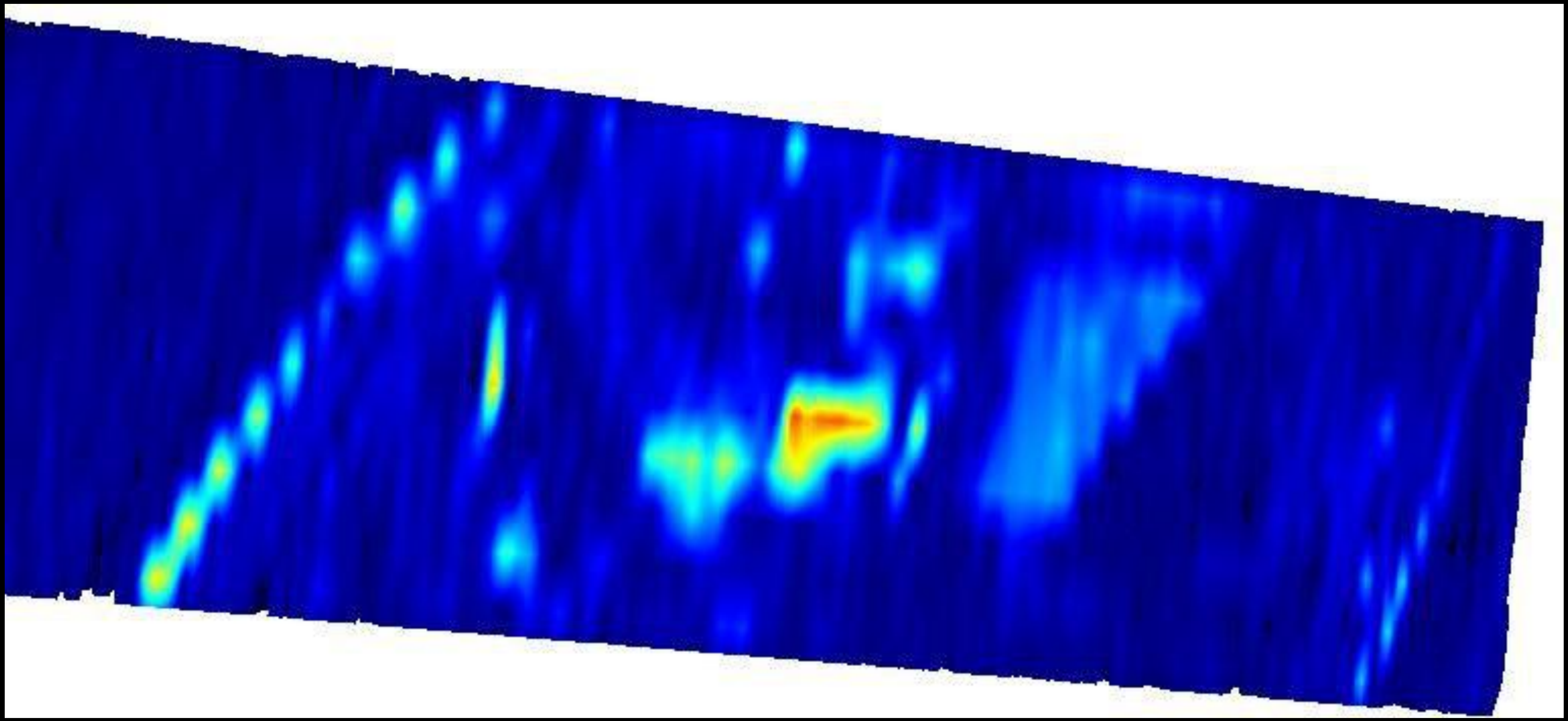
Interaction – Phantom Omni

Interaction by proxy

- Plane estimation on surface
- Constant re-evaluation
- Real-time Performance

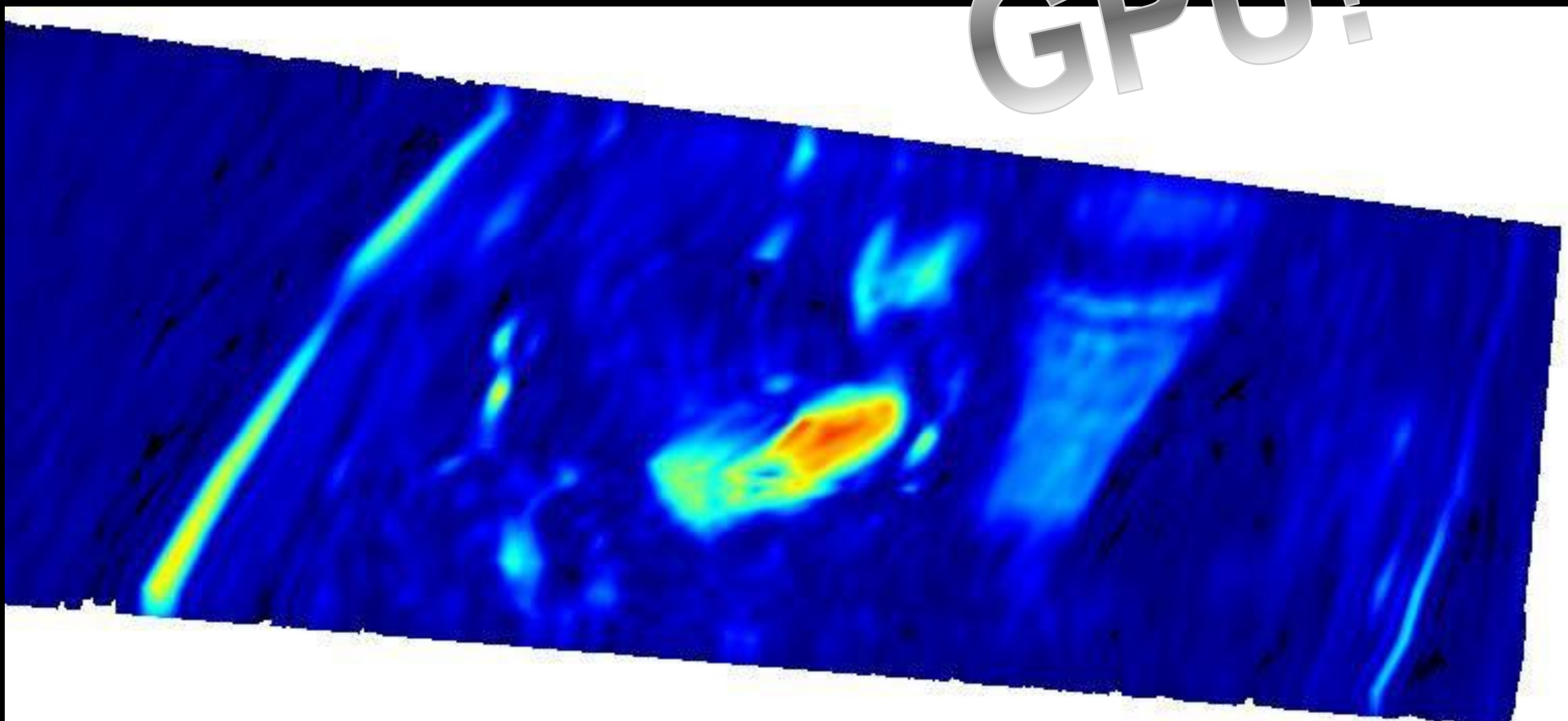


Linear Interpolation



Registration Based Interpolation

GPU!



Motion compensation

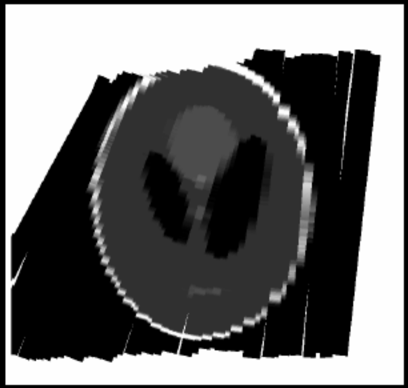
- Phantom

- Motion stack 1

- Motion stack 2

- Low resolution

- Comp. stack 1



- Comp. stack 2

Conclusion

- We have demonstrated that
- Iterative reconstruction using smart priors allows for
- Image reconstruction using fewer projections at low resolution, and
- Motion compensation

Acknowledgements

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