Modelling of Heat and Mass Transfer in Food Products

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Abstract: The use of the finite-element method for understanding and analyzing the freezing and drying processes of food products is in focus in this paper. The objective of this study is to develop a model that can predict temperature distribution and weight loss of food products during the freezing and drying processes.

The problem was solved by utilizing heat, mass transfer and moving mesh model. In order to predict the heat transfer, the thermo physical properties of food products are evaluated by a composition-based prediction method where the thermal properties of a given food product are evaluated based on major components found in the food. The mass transfer problem in the unfrozen region is solved by the Fickian model and in the frozen region with the receding front model that assumes that the water movement becomes immobilized and that the moisture sublimates first from the surface then through the dried area in the food.

From the model the temperature, moisture and evaporation rates are obtained as a function of location and time.

Keywords: Food processing, food freezing, food drying, mass transfer, heat transfer.

1. Introduction

Food drying and food freezing are the most common and widely used methods to preserve food for later use. Nevertheless, the processing of food is complex and still little understood. The processing and storage conditions of the food products are highly related to the product end quality and also to economic factors.

The main objective of this study is to develop a general model that can account for the food characteristics, the processing and storage conditions on the freezing and drying times of the food products. That is done by applying some of the methods suggested in the literature.

2. Methods

2.1 Thermal properties of food

Knowledge of the thermal properties of food is required in order to evaluate the heat transfer in food products. Therefore, a composition-based prediction method was used to obtain the thermal properties of food. The thermal properties are described by major components found in food such as: proteins, fat, carbohydrate, fibers, ash and water/ice. The composition data is stated as mass fraction and given in [1] and [2] for different food products such as vegetables, fruits, meat, etc. The thermo physical properties of each component are subsequently described by temperature dependent mathematical models in the temperature range of -40°C to 150°C. The properties were evaluated in the global expression window in Comsol.

Knowing the composition as well as the thermal properties of the different components makes it possible to find the apparent properties of the food product. A different approach is used for the evaluation of the properties in the unfrozen and frozen region of the food. In order to evaluate which approach is to be used, determination of the initial freezing point is required. Pham Q.T. [3] has suggested the following equation for evaluation of the initial freezing point based on the composition data of the product:

$$T_f = -0.141 - 4.36\frac{X_o}{X_{wo}} - 43.5\frac{X_a}{X_{wo}} \quad (1)$$

Whether the temperature of the product is above or below the initial freezing point the thermal properties of the food are calculated according to one of the following approaches:

Nomenclature			
X	Mass fraction [%]	k_c	Mass transfer coefficient [m / s]
X_o	Mass fraction of all solid components (X_p, X_{ft}, X_c, X_{fb}) [%]	$\dot{m}$	Mass flux [mol / (m^2 s)]
ρ	Density [kg / m^3]	ε	Porosity [-]
Cp	Specific heat capacity [J / (kg K)]	D_Y	Diffusion coefficient of vapor through dried section of food [m^2 / s]
λ	Conductivity [W / (m K)]	D_{eff}	Effective diffusion coefficient [m^2 / s]
T	Temperature [K]	**Subscripts**	
L_f	Latent heat of fusion [kJ / kg]	ice	Ice
T_f	Initial freezing temperature [K]	wo	Water
h	Convective heat transfer coefficient [W / (m^2 K)]	p	Proteins
Q_{evap}	Heat flux [W / m^2]	ft	Fat
Q_{sub}	Heat flux [W / m^2]	fb	Fiber
q	Heat flux [W / m^2]	a	Ash
δ	Thickness of dried layer [m]	c	Carbohydrates
C	Moisture concentration [mol / m^3]		

Unfrozen food ($T > T_f$)

The density of the food product is calculated according to the following equation:

$$\rho = \frac{1-\varepsilon}{\sum \frac{X_i}{\rho_i}} \qquad (2)$$

The specific heat capacity in the unfrozen part of the food is obtained from the mass average of the heat capacities of the components in the food product:

$$Cp_w = \sum Cp_i X_i \qquad (3)$$

The thermal conductivity is described by means of a Maxwell type model. The dispersed phase is air (if present) and the continuous phase in the equation is obtained from the mass average of the conductivities of the remaining components in the food product.

$$\lambda = \lambda_c \frac{2\lambda_c + \lambda_d - 2\varepsilon(\lambda_c - \lambda_d)}{\lambda k_c + \lambda_d + \varepsilon(\lambda_c - \lambda_d)} \qquad (4)$$

Frozen food ($T \le T_f$)

The ice fraction during freezing is determined by the following equation given by [1]:

$$X_{ice} = \frac{1.105 * X_{wo}}{1 + \frac{0.7138}{\ln(T_f - T + 1)}} \qquad (5)$$

Knowing the fraction of ice, the thermal properties of the food during freezing can be obtained. The density of the food product is calculated according to equation 2, however, now including the mass fraction of ice.

The latent heat of fusion is accommodated in the description of the apparent specific heat of food during freezing.

$$Cp_w = \sum Cp_i X_i - L_f \frac{\partial X_{ice}}{\partial T} \qquad (6)$$

Thermal conductivity is determined in two steps and is based on equation 4. First, ice is used as a dispersed phase (if present). Secondly, air is used as dispersed phase (if present) and continuous phase is the previously obtained conductivity.

The thermal properties are evaluated in the global expression window in Comsol.

2.2 Moisture loss by evaporation or sublimation

Two approaches are used when the temperature of the product is above or below the initial freezing point.

Unfrozen food ($T > T_f$)

During drying, the mass transfer of water is described by means of Fick's law, [6] and [7]. The water diffuses to the surface of the food products where it evaporates. The heat transfer is evaluated by means of Fourier's law. Evaporation energy is taken into consideration at the boundary in the energy equation as Q_{evap}.

Governing equations:

$$\frac{\partial C}{\partial t} = \nabla D_{eff} \nabla C \tag{7}$$

$$\rho Cp \frac{\partial T}{\partial t} = \nabla \lambda \nabla T \tag{8}$$

Boundary conditions:

$$-n \cdot (-D\nabla C) = k_c (C_{out} - C_s) \tag{9}$$

$$-n \cdot (-\lambda \nabla T) = h(T_{out} - T_s) - Q_{evap} \tag{10}$$

Frozen food ($T \leq T_f$)

Once the food is frozen, the water movement inside the dense food products becomes immobilized and the moisture sublimates first from surface and then through the dried area in the food [6] and [7], see figure 1.

Figure 1: Sketch of model – frozen food.

This approach was modeled by coupling the heat transfer module with the moving mesh module in Comsol. The mass flux is expressed by [6]:

$$\dot{m} = \frac{(C_{out} - C_s)}{\frac{1}{k_c} + \frac{\delta}{D_Y}} \tag{11}$$

And the speed of the receding sublimating front is expressed as [6]:

$$\frac{\partial \delta}{\partial t} = \frac{\dot{m}}{C(\delta)} \tag{12}$$

At the sublimating boundary the heat is transferred throughout the layer of dried food:

$$-n \cdot (-\lambda \nabla T) = q - Q_{sub} \tag{13}$$

$$q = \frac{(T_{out} - T_{sub})}{\frac{1}{h} + \frac{\delta}{\lambda}} \tag{14}$$

2.3 Model setup

Three products were investigated: two vegetables (potato and carrot) and one meat product (lamb). The vegetables were exposed to a convective hot air drying process while the meat was frozen and the weight loss during storage was estimated.

Some spreading of the values for the diffusion coefficient was found in the literature and therefore it was chosen to proceed with the findings in [4] for the vegetables. Here a cylindrical sample of the food was taken with a diameter of Ø 7mm. In the Comsol model, the cylindrical sample was simplified to a 2 dimensional case taking a cross sectional area of the cylindrical sample as a point of departure.

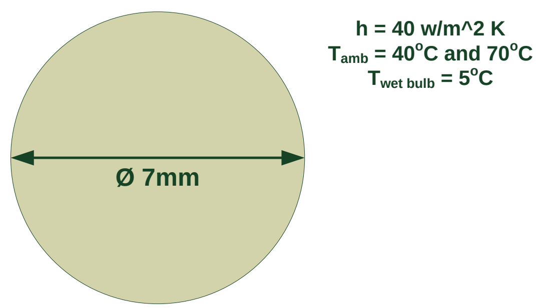

Figure 2: Sketch of model – vegetables.

Regarding the meat product, the freezing time and weight loss during storage were investigated with a 1D model. That was done to lower the computational time. The diffusion coefficient was found in [7], however, deviations were also found. The model of the lamb can be seen in the figure below.

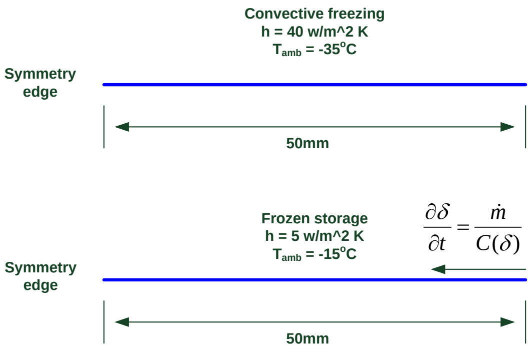

Figure 3: Sketch of model – meat.

3. Results and discussion

3.1 Convective hot air drying (coupled heat and mass transfer module)

The vegetable samples were exposed to convective drying at two different temperatures. The results are presented in figure 4-7 where the drying curves and center temperature are plotted as a function of time. Higher temperature results in a faster dehydration of the food.

From the drying curve two distinct regions can be depicted:

- a constant drying rate period where the surface remains saturated
- a falling drying rate period where internal water diffusion is the limiting factor in the food dehydration.

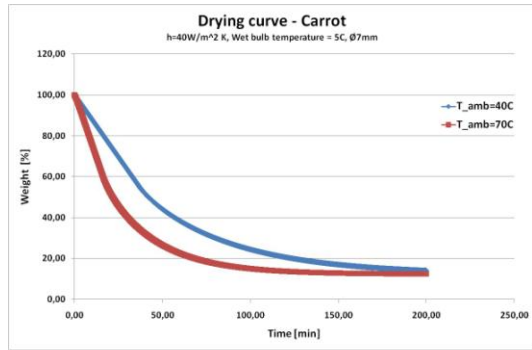

Figure 4: Drying curve – carrot.

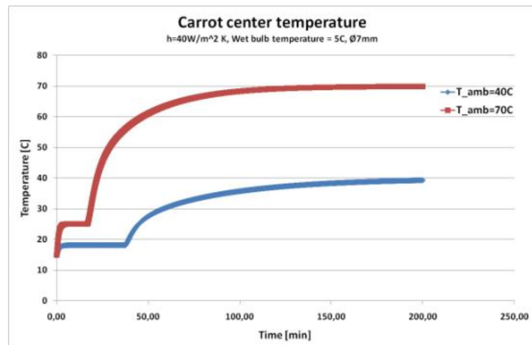

Figure 5: Center temperature – carrot.

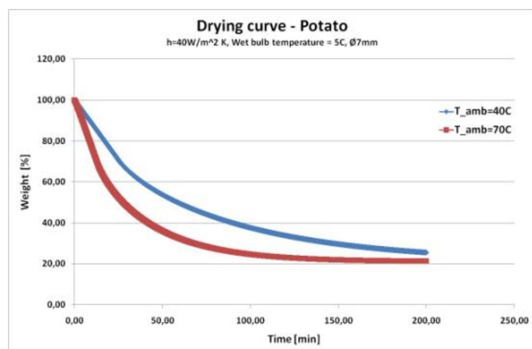

Figure 6: Drying curve – potato.

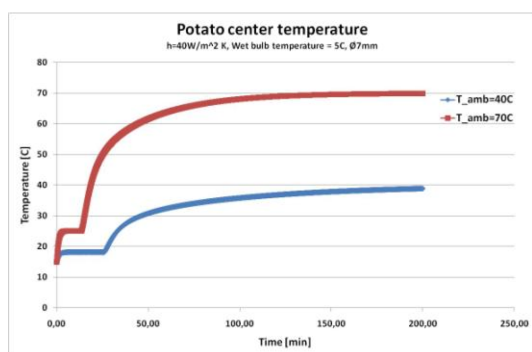

Figure 7: Center temperature – potato.

3.2 Convective freezing (heat transfer module)

The lamb was exposed to freezing at an air temperature of -35^{o}C. The results are presented in figure 8 showing temperature distribution at different locations in the lamb as a function of time.

3.3 Frozen storage (coupled heat and moving mesh module)

The lamb is simulated to be stored in a cold store at a temperature of -15^{o}C. The weight loss is plotted in figure 9 as a function of time, and the thickness of the desiccated region in the food is plotted in figure 10. From the figures it can be seen that as the layer of the desiccated region in the food becomes thicker, the mass flux becomes lower.

4. Conclusions

The developed model allows for the evaluation of freezing and sublimation fronts, freezing times, temperature profiles and weight loss during storage of frozen food. It can also be used to evaluate the weight loss during drying of the food products.

However, in order to have a general model that can account for the food characteristics and the processing and storage conditions it is necessary to carry out additional work on the moisture migration in food products.

While the thermal properties of the food seem to be well- described for a large amount of the food products, the data on moisture diffusivity are hard to find. During the study it was difficult to obtain data on the effective diffusion coefficient and the values found in the literature show some spreading.

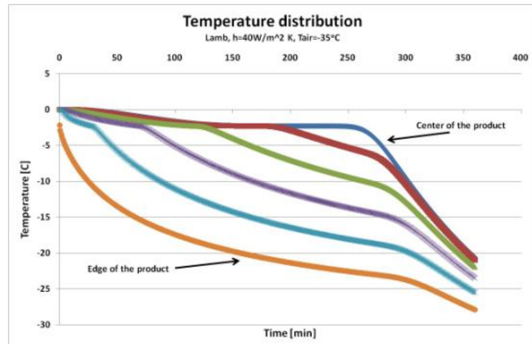

Figure 8: Temperature distribution in lamb during freezing.

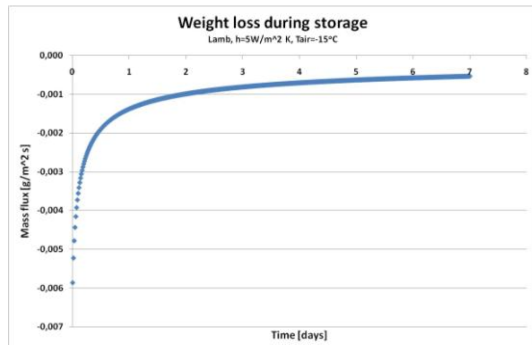

Figure 9: Weight loss during storage.

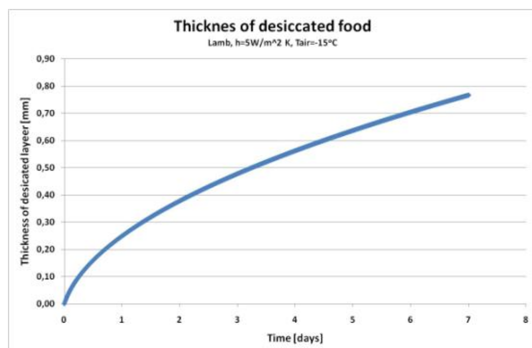

Figure 10: Thickness of desiccated layer.

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6. Appendix A

	Potato	Carrot	Lamb
Water X_{wo}	78,96	87,77	73,37
Proteins X_p	2,07	1,03	20,29
Fat X_{ft}	0,1	0,19	5,28
Carbohydrate X_c	16,38	7,14	0,0
Fibre X_{fb}	1,6	3,0	0,0
Ash X_a	0,89	0,87	1,06

7. Appendix B

Product	T [C]	D [m^2/s]	Reference
Potato	31	1,60 e^{-10}	[10]
	40	4,68 e^{-10}	[4]
	50	5,87 e^{-10}	[4]
	50	4,80 e^{-10}	[10]
	54	2,58 e^{-10}	[10]
	54	0,258 e^{-10}	[8]
	60	0,394 e^{-10}	[8]
	60	6,20 e^{-10}	[4]
	60	8,00 e^{-10}	[4]
	65	2,00 e^{-10}	[10]
	65,5	0,437 e^{-10}	[8]
	68,8	0,636 e^{-10}	[8]
	70	6,36 e^{-10}	[10]
	70	8,10 e^{-10}	[10]
	70	10,2 e^{-10}	[4]
Carrot cubes	40	0,675 e^{-10}	[8]
	60	1,21 e^{-10}	[8]
	80	1,79 e^{-10}	[8]
	100	2,41 e^{-10}	[8]
Carrot	50	5,56 e^{-10}	[10]
	60	7,48 e^{-10}	[10]
	70	9,89 e^{-10}	[10]
Carrot (core)	40	6,42 e^{-10}	[4]
	50	8,63 e^{-10}	[4]
	60	11,5 e^{-10}	[4]
	70	14,7 e^{-10}	[4]
Carrot (cortex)	40	6,68 e^{-10}	[4]
	50	8,49 e^{-10}	[4]
	60	9,71 e^{-10}	[4]
	70	11,83 e^{-10}	[4]
Frozen lamb	-15	0,3 e^{-6} 3,3 e^{-6}	[7]