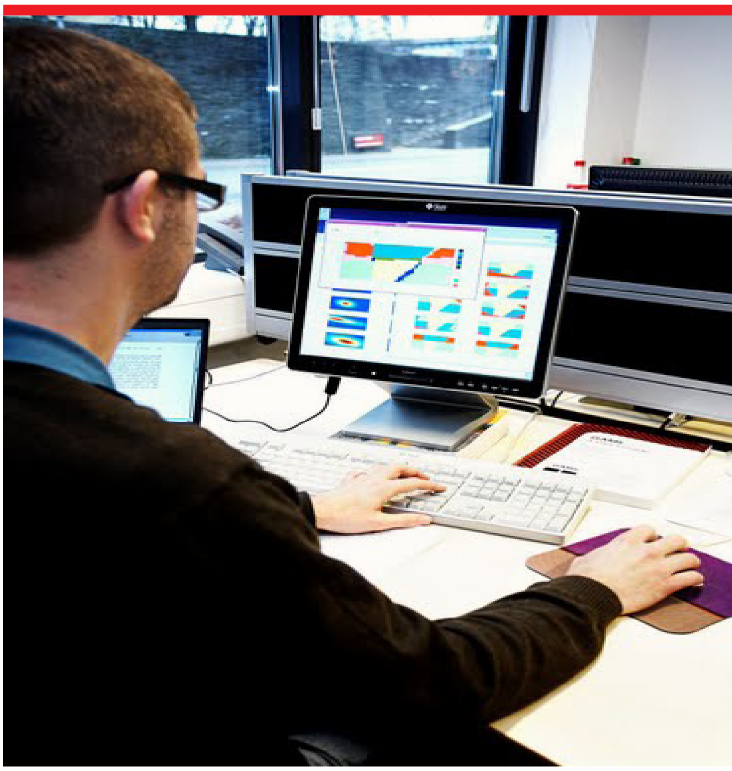
Raw material utilization in slaugter-houses – optimizing expected profit using mixed-integer programming



Report 8. 2010

DTU Management Engineering

Toke Koldborg Jensen
Niels Kjærsgaard

April 2010

DTU Management Engineering
Department of Management Engineering

Raw material utilization in slaughterhouses – optimizing expected profit using mixed-integer programming

Toke Koldborg Jensen[a,*] and Niels Kjærsgaard[b]

[a] *Department of Management Engineering, Technical University of Denmark, Produktionstorvet, Building 426, 2800 Kgs. Lyngby, Denmark*
[b] *Danish Meat Research Institute, division of Danish Technological Institute, Maglegårdsvej 2, 4000 Roskilde, Denmark*

Abstract

Slaughterhouses are major players in the pork supply chain, and supply and demand must be matched in order to generate the highest profit. In particular, carcasses must be sorted in order to produce the "right" final products from the "right" carcasses. We develop a mixed-integer programming (MIP) model for computing the optimal sorting of carcasses according to two parameters; slaughter weight and fat layer. Moreover, we consider a new approach for dealing with expected measurement errors. The results provide insight into how sorting groups should be designed in order to improve the profit at slaughterhouses. Finally, we comment on the expected effect of variations in the raw material supply and the demand as well as future research concerning joint modelling of supply chain aspects.

Keywords: Mixed-integer programming, optimization, pork production, sorting.

1 Introduction

The paper is concerned with pork production, and in particular with the sorting of pig carcasses after slaughtering at Danish state-of-the-art slaughterhouses. A main characteristic of the Danish pork sector is that the slaughterhouses are co-operatively owned by the primary producers and that they are required to receive the delivered pigs. This results in a strong push of raw materials. On the other hand, strong markets-side actors have specific requirements for the delivered products both concerning quality and quantities. This results in a strong pull of products. The main decoupling point is located within the slaughterhouse where the slaughtered pigs must be balanced with the demand for final products. The sector also maintains general breeding programs which result in relatively homogeneous pigs. However, natural variations, different feed and growing conditions as well as different management schemes on the pig farms still result in considerable variations in the supply.

Figure 1 schematically illustrates the supply chain around and within the slaughterhouses. In the slaughterhouses, the pigs are slaughtered and the carcasses are placed in a cold storage room to cool down before they are further processed into final products – typically the following day. The raw material variation and the varying specifications for different final products mean that

*Corresponding author: tkje@man.dtu.dk (T.K. Jensen)

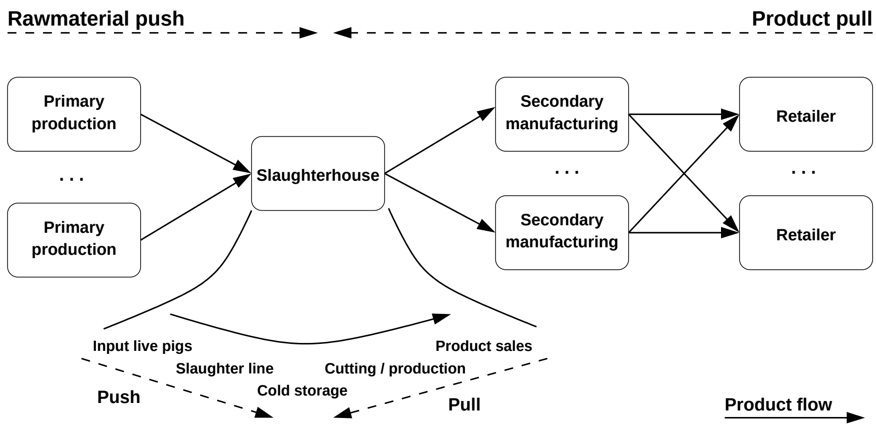


Figure 1: Schematic illustration of simple pork supply chain showing the overall chain from primary production to retailers, as well as the main chain within the slaughterhouse.

not all carcasses are equally suited for the production of any mix of final products. The "right" carcasses must be used for the "right" final products in order to optimize the overall profit. However, individual handling of each carcass is not possible due to space and handling limitations, and the carcasses must be sorted into a limited number of sorting groups. The sorting is therefore central in order to maximize the profit. In practice, the construction of the sorting groups is based on parameters describing the carcasses; e.g. fat layer, lean meat percentage and slaughter weight, such that the carcasses in a given sorting group are believed to be good for a given product mix. However, the currently used sorting groups may have overlapping sorting criteria, and it is difficult to estimate the effects of raw material variations, demand requirements and whether or not the sorting groups are at all optimal.

We propose a mixed-integer programming (MIP) model for computing theoretically optimal sorting groups based on slaughter weights and fat layers of the carcasses. The MIP model builds on a distribution of carcasses over a grid spanned by the two parameters, and includes a new approach to handle measurement errors. The central part of the paper is a joint modeling of the numerical application of measurement errors and an MIP model for optimizing the sorting groups. Models based on the one presented here are relevant for the pork sector to gain new insight into the optimal sorting of pig carcasses in order to improve the profit.

2 Optimization models in pork production

Mathematical models are widely used for analysis of food supply chains, e.g. for devising optimal production plans; [4], and for modeling distribution networks; [2]. Food related aspects such as limited shelf-life, traceability, and the perishable nature of raw materials are also addressed; see e.g. [3] and [11]. The interaction between the steps of the supply chain is a main issue in supply chain management and models are also developed to support coordination; see e.g.

[5] where Gribkovskaya *et al.,* formulate an MIP model which combines livestock collection and transportation with production planning. Customer and consumer demands are also relevant for supply chains, and optimization models describing the retail end of the supply chains are also found in the literature; see e.g. [10].

Within pork supply chains, research has been done in areas such as facility location; [1], transportation and distribution planning; [9], and marketing planning for primary producers; [8]. However, the optimal use of carcasses within the slaughterhouses is less studied – especially considering quality aspects and the suitability of using particular carcasses for particular final products. Nevertheless, raw material costs constitute the major cost for the slaughterhouses, and even small improvements in raw material usage will result in large benefits for the slaughterhouses; see the Ph.D thesis [7].

An MIP model for computing the optimal use of specific predefined groups of carcasses was developed by Kjærsgaard in [6]. The model is based on slaughter weights and fat layer parameters and measurement errors are included through simulated measurement noise. The MIP model in the current paper is based on the same basic ideas and a similar production yield model. However, the developed model is based on a discretization of the parameters describing the raw materials and includes an innovative explicit modeling of measurement errors. Furthermore, predefined sorting groups can be specified while still computing the optimal use of each of these groups. The developed model also supports the direct inclusion of demand constraints even though this is not studied in the current paper.

3 Sorting strategies and measurement errors

A carcass can be used for one of many products mixes M_n, $n \in \{1, 2, \ldots, N\}$. Each product mix consists of a combination of products obtained from the fore-end, middle piece and back-end of the pig carcass resulting in a number of final products. We assume that the parameters fat layer f and slaughter weight w are used for sorting the carcasses into different sorting groups, and that carcasses with identical f and w are used for the same product mix. Then the binary functions:

$$S_{M_n}(f, w) = \begin{cases} 1 & \text{if carcasses with } (f, w) \text{ are used for } M_n \\ 0 & \text{otherwise.} \end{cases} , \tag{1}$$

describe for each M_n a two-dimensional map of the raw materials to use for the product mix. Exactly one of the functions S_{M_n} for $n \in \{1,\, 2,\, \ldots,\, N\}$ is required to be one, i.e.:

$$\sum_{n=1}^{N} S_{M_n}(f, w) = 1, \quad \forall\, (f, w), \tag{2}$$

which ensures that all carcasses are used. The profit $\mathcal{P}_{M_n}(f, w)$ obtained when producing product mix M_n from raw materials with parameters (f, w) can be estimated using the product yield model given in Appendix A.

In practice the carcasses are sorted into a number of sorting groups $g \in \{1, 2, \ldots, G\}$ due to limitations in the cold storage room. We assume that each

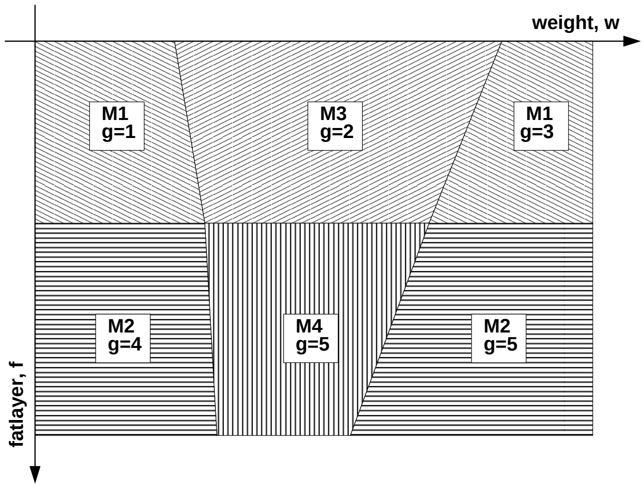


Figure 2: Illustration of two-dimensional domain spanned by weight and fat layer, six sorting groups ($G = 6$) and production of four product mixes M_1 to M_4.

sorting group covers a unique area in the two-dimensional domain spanned by f and w. Moreover, it is assumed that all carcasses in a sorting group g are used for the same product mix. We define the functions:

$$F(f, w) \in \{1, 2, \ldots, G\} \tag{3}$$

$$U(g) \in \{1, 2, \ldots, N\} \tag{4}$$

such that $F(f, w) = g$ if carcasses with measured parameters (f, w) belong to sorting group g, and $U(g) = n$ if product mix M_n is produced from raw materials in sorting group g. We therefore have the relation:

$$U(g) = nS_{M_n}(f, w), \quad \forall\, (f, w) \text{ such that } F(f, w) = g, \tag{5}$$

which states that the binary function $S_{M_n}(f, w)$ is one throughout the area covered by sorting group g if product mix M_n is to be produced from raw materials in group g. Optimal production is obtained by designing the sorting groups such that the optimal product mix is produced from the raw materials. A schematic example of six sorting groups is shown in Figure 2. The example shows how product mix M_1 is optimally produced from carcasses in sorting groups $g \in \{1, 3\}$, product mix M_3 is optimally produced from carcasses in sorting group $g = 2$, etc.

In practice weights and fat layers are measured and therefore affected by measurement errors such that some carcasses are erroneously placed in adjacent sorting groups. This leads to non-optimal production and therefore loss of profit. We assume that the measurement errors are normally distributed with zero mean and standard deviation σ_f and σ_w for the fat layer and weight, respectively. We also assume that there is no correlation between measurement errors for fat layer and weight such that:

$$\mathcal{G}(f, w) = \frac{1}{2\pi\sigma_f\sigma_w} e^{-\left(\frac{f^2}{2\sigma_f^2} + \frac{w^2}{2\sigma_w^2}\right)} \tag{6}$$

describes the measurement errors. Then $\mathcal{G}(f - \hat{f}, w - \hat{w})$ is the probability that the true parameters f and w are measured as $\hat{f}$ and $\hat{w}$. Given the binary sorting groups $S_{M_n}(f, w)$ we can by convolution find the probability that a given product mix is produced from raw materials with true parameters (f, w) as:

$$P_{M_n}(f, w) = (S_{M_n} * \mathcal{G})(f, w) \tag{7}$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_{M_n}(\hat{f}, \hat{w}) \mathcal{G}(f - \hat{f}, w - \hat{w}) d\hat{f} d\hat{w}. \tag{8}$$

3.1 Raw material distribution and production

The carcasses are not uniformly distributed over the domain. Moreover, heavy carcasses often have a thick fat layer, whereas lighter carcasses have a thin fat layer. This is described by the probability distribution $D(f, w)$ representing the probability that a carcass has a given combination of fat layer and weight. Assuming that the total number of pigs considered is R then the expected number of carcasses to be used for each product mix is:

$$c_{M_n}(f, w) = P_{M_n}(f, w) D(f, w) R. \tag{9}$$

The expected profit of using a particular set of sorting groups for certain product mixes can be computed as:

$$\mathcal{O} = \sum_{n=1}^{N} \int_f \int_w c_{M_n}(f, w) \mathcal{P}_{M_n}(f, w) df dw, \tag{10}$$

where $\mathcal{P}_{M_n}(f, w)$ is the expected profit obtained by producing product mix M_n from a carcass with parameters (f, w) as computed by the product yield model in Appendix A. Similarly, the estimated production y of each final product j described by the product yield model can be computed as:

$$y_j = \sum_{n \in \mathcal{M}_j} \int_f \int_w c_{M_n}(f, w) W_j(f, w), \tag{11}$$

where $\mathcal{M}_j$ is the set of all product mixes M_n from which the final product j is produced, and $W_j(f, w)$ is the estimated product weight as defined in Appendix A. Demand constraints can be formulated by restricting y_j.

3.2 True and measured parameters

The model is at the same time based on assumed true values of the parameters and measured values due to the integrated convolution of the sorting groups. The raw material distributions and the profit calculations are based on assumed true values, whereas the sorting groups are based on measured values; see Figure 3. Working in both domains simultaneously allows us to formulate constraints both on the sorting groups in the measured domain, and on the production based on assumed true final product yields. Eq. (5) is an example on how the sorting groups may be restricted, and a demand constraint will be an example of a constraint in the true parameter domain.

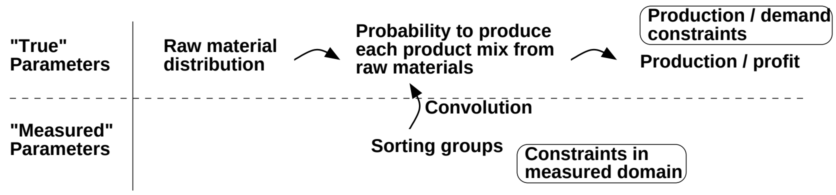


Figure 3: Illustration of model operating in two domains; one based on assumed true parameters, and one based on measured parameters.

4 Discretization and mathematical programming

The optimal use of the carcasses was in [6] illustrated on a two-dimensional grid spanned by fat layer and slaughter weight. However, the grid was not an integrated part of the optimization model. A similar discretization using F fat layer groups and W weight groups resulting in $F \times W$ cells is here used as the basis for the optimization model itself; see Figure 4. The discretization step sizes are defined by Δ_f and Δ_w, and $f_{\min}$, $f_{\max}$, $w_{\min}$ and $w_{\max}$ define the fat layer and weight in the center of the first and last cells of the domain. For simplicity, we assume that the parameters are selected such that $\Delta_f = 1\,\text{mm}$ and $\Delta_w = 1\,\text{kg}$. These choices comply with the illustrations of results in [6]. Each cell covers an interval around the center value such that the cell defined by $(f_\mathbf{f}, w_\mathbf{w})$ covers the intervals $[f_\mathbf{f} - \frac{1}{2}; f_\mathbf{f} + \frac{1}{2}[$ and $[w_\mathbf{w} - \frac{1}{2}; w_\mathbf{w} + \frac{1}{2}[$. The calculation of product yield and estimated profit for each cell is by default based on the center of the cell. That is, all carcasses falling within each cell are implicitly assumed to have parameters corresponding to the center of the cell. This leads to discretization errors in the representation of the assumed true raw material distribution. However, in practice the true raw material distribution is not known, and the discrete distribution represents one possible distribution based on the assumed true distribution. Moreover, the true product yield and obtained profit may deviate from the estimated product yield and profit based on the linear product yield model in Appendix A. A simulation study in Section 5 is concerned with sorting of the true raw materials based on sorting groups generated by the proposed model. Further analyses of varying raw material distributions, demand requirements and better product yield models are part of current research, and outside the scope of the present paper.

4.1 The MIP model

The MIP model is based on a number of sets, parameters, decision variables and constraints. Boldface characters are used to indicate the discrete elements of the sets given below, and the parameters and decision variables are given in Tables 1 and 2, respectively.

Fat layer indices: $\mathbf{f} \in \{1, 2, \ldots, F\}$ such that the fat layer in the center of the cells is given as $f_\mathbf{f} = f_{\min} + (\mathbf{f} - 1)$.

Weight indices: $\mathbf{w} \in \{1, 2, \ldots, W\}$ such that the weight in the center of the cells is given as $w_\mathbf{w} = w_{\min} + (\mathbf{w} - 1)$.

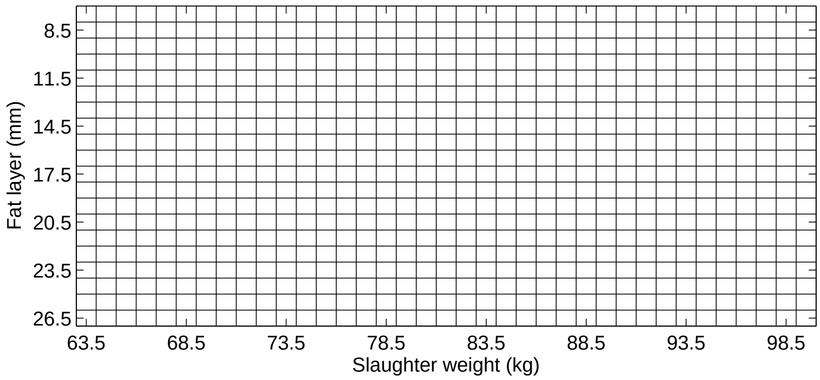


Figure 4: Illustration of discrete parameter space.

Product mix indices: $\mathbf{n} \in \{1, 2, \ldots, N\}$.

Final products: $\mathbf{j} \in \{1, 2, \ldots, J\}$.

Sorting groups: $\mathbf{g} \in \{1, 2, \ldots, G\}$.

Bandwidth: $\mathbf{b}_f, \mathbf{b}_w \in \{1, 2, \ldots, B\}$, (see below).

Constraints

All cells are assigned to a sorting group, which is used for exactly one of the N product mixes:

$$\sum_{\mathbf{n}} s_{\mathbf{n},\mathbf{f},\mathbf{w}} = 1, \quad \forall\, \mathbf{f}, \mathbf{w}. \tag{12}$$

For each of the sorting groups $\mathbf{g}$, we have

$$u_{\mathbf{g}} = \sum_{\mathbf{n}} \mathbf{n} s_{\mathbf{n},\mathbf{f},\mathbf{w}}, \quad \forall\, \mathbf{f}, \mathbf{w} \text{ such that } \mathrm{FIX}_{\mathbf{f},\mathbf{w}} = \mathbf{g}, \quad \forall\, \mathbf{g} \tag{13}$$

which is the discrete version of Eq. (5) ensuring that all cells belonging to the fixed sorting group $\mathbf{g}$ are used for the same product mix.

A discrete blurring operator, $\mathrm{H}_{\mathbf{b}_f,\mathbf{b}_w}$, based on Eq. (6) is defined in Appendix B. We assume a discretization over the domain 1 to B, where the bandwidth B is uneven such that $h = \left\lfloor \frac{B}{2} \right\rfloor$ defines the half bandwidth, and $m = h + 1$ defines the midpoint. The convolutions of $\mathrm{H}_{\mathbf{b}_f,\mathbf{b}_w}$ with the binary maps defined by the decision variables $s_{\mathbf{n},\mathbf{f},\mathbf{w}}$ result in associated probability maps:

$$p_{\mathbf{n},\mathbf{f},\mathbf{w}} = \sum_{\hat{\mathbf{f}}=\mathbf{f}-h,\, \hat{\mathbf{w}}=\mathbf{w}-h}^{\hat{\mathbf{f}}=\mathbf{f}+h,\, \hat{\mathbf{w}}=\mathbf{w}+h} \mathrm{H}_{\mathbf{f}-\hat{\mathbf{f}}+m,\mathbf{w}-\hat{\mathbf{w}}+m} s_{\mathbf{n},\hat{\mathbf{f}},\hat{\mathbf{w}}}, \quad \forall\, \mathbf{n}, \mathbf{f}, \mathbf{w}, \tag{14}$$

describing the probability to use raw materials with parameters $f_{\mathbf{f}}$ and $w_{\mathbf{w}}$ for product mix $\mathbf{n}$ given the sorting groups defined by $s_{\mathbf{n},\mathbf{f},\mathbf{w}}$. However, $\hat{\mathbf{f}}$ and $\hat{\mathbf{w}}$ in Eq. (14) fall outside the domain for values of $\mathbf{f}$ and $\mathbf{w}$ close to the boundary. This

Parameter	Description
$\mathrm{H}_{\mathbf{b}_f,\mathbf{b}_w}$	A discrete blurring operator based on Eq. (6) that define the measurement errors.
$\mathrm{H}^M_{\mathbf{f},\mathbf{w},\hat{\mathbf{f}},\hat{\mathbf{w}}}$	A precomputed parameter relating true and measured values. See below.
$\mathrm{PROFIT}_{\mathbf{n},\mathbf{f},\mathbf{w}}$	The estimated profit obtained when producing each of the product alternatives from raw materials with parameters $f_{\mathbf{f}}$ and $w_{\mathbf{w}}$ using the product yield model from Appendix A.
$\mathrm{WEIGHT}_{\mathbf{j},\mathbf{f},\mathbf{w}}$	The estimated weight of final product $\mathbf{j}$ if produced from raw materials with parameters $f_{\mathbf{f}}$ and $f_{\mathbf{w}}$ using the product yield model from Appendix A.
$\mathrm{USE}_{\mathbf{j},\mathbf{n}}$	Binary parameter taking value one if the final product $\mathbf{j}$ is produced from product mix n and zero otherwise.
$\mathrm{FIX}_{\mathbf{f},\mathbf{w}}$	The predefined group number that a carcass with parameters $f_{\mathbf{f}}$ and $w_{\mathbf{w}}$ belongs to.
$\mathrm{D}_{\mathbf{f},\mathbf{w}}$	Distribution of raw material.
R	Total number of pig carcasses.

Table 1: Parameters.

Decision variables	Description
$s_{\mathbf{n},\mathbf{f},\mathbf{w}}$	Binary variables describing the sorting groups from Eq. (1) over the discrete domain. These variables form binary images of the sorting groups for each product mix $\mathbf{n}$.
$u_{\mathbf{g}}$	The product alternative to produce from sorting group $\mathbf{g}$ as defined in Eq. (4).

Table 2: Decision variables.

results in the implicit assumption that no sorting groups are used beyond the boundaries because values outside the domain are by default assumed to be zero. A better assumption is that the sorting groups at the boundary are continued also outside the boundary. For simplicity, we only consider measurement errors in the fat layer parameters, and hence continued sorting groups along the sides of the domain where $\mathbf{f} = 1$ and $\mathbf{f} = F$. The convolutions with these boundary conditions along the two sides of the domain are given as:

$$
\begin{aligned}
p^C_{\mathbf{n},\mathbf{f},\mathbf{w}} &= p_{\mathbf{n},\mathbf{f},\mathbf{w}} \\
&+ \sum_{\hat{\mathbf{f}}=0,\,\hat{\mathbf{w}}=\mathbf{w}-h}^{\hat{\mathbf{f}}=h-\mathbf{f}+1,\,\hat{\mathbf{w}}=\mathbf{w}+h} \mathrm{H}_{m+1-\hat{\mathbf{f}}-\mathbf{f},\mathbf{w}-\hat{\mathbf{w}}+m} s_{\mathbf{n},1,\hat{\mathbf{w}}} \\
&+ \sum_{\hat{\mathbf{f}}=2F-\mathbf{f}-h+1,\,\hat{\mathbf{w}}=\mathbf{w}-h}^{\hat{\mathbf{f}}=F+1,\,\hat{\mathbf{w}}=\mathbf{w}+h} \mathrm{H}_{m+1-\hat{\mathbf{f}}-\mathbf{f},\mathbf{w}-\hat{\mathbf{w}}+m} s_{\mathbf{n},F,\hat{\mathbf{w}}}, \quad \forall \mathbf{n},\mathbf{f},\mathbf{w}, \qquad (15)
\end{aligned}
$$

where the superscript C indicates continuous sorting groups. A parameter $\mathrm{H}^M_{\mathbf{f},\mathbf{w},\hat{\mathbf{f}},\hat{\mathbf{w}}}$ which explicitly relates each combination of true and measured parameters can be pre-computed based on $\mathrm{H}_{\mathbf{b}_f,\mathbf{b}_w}$, and Eq. (15) is simplified to:

$$p^C_{\mathbf{n},\mathbf{f},\mathbf{w}} = \sum_{\hat{\mathbf{f}},\hat{\mathbf{w}}} \mathrm{H}^M_{\mathbf{f},\mathbf{w},\hat{\mathbf{f}},\hat{\mathbf{w}}} s_{\mathbf{n},\hat{\mathbf{f}},\hat{\mathbf{w}}}, \quad \forall \mathbf{n},\mathbf{f},\mathbf{w}. \qquad (16)$$

The expected number of carcasses used for each product mix is given by:

$$c_{\mathbf{n},\mathbf{f},\mathbf{w}} = \mathrm{D}_{\mathbf{f},\mathbf{w}} \mathrm{R} p^C_{\mathbf{n},\mathbf{r},\mathbf{w}}, \quad \forall \mathbf{n},\mathbf{f},\mathbf{w}, \qquad (17)$$

in analogy with Eq. (9), and the amount of final products can be computed as:

$$y_{\mathbf{j}} = \sum_{\mathbf{n},\mathbf{f},\mathbf{w}} \mathrm{USE}_{\mathbf{j},\mathbf{n}} \mathrm{WEIGHT}_{\mathbf{j},\mathbf{f},\mathbf{w}} c_{\mathbf{n},\mathbf{f},\mathbf{w}} \quad \forall \mathbf{j}. \qquad (18)$$

Demand constraints can then eventually be formulated by restricting $y_{\mathbf{j}}$. Finally, the objective is to select the optimal use of the raw materials in each sorting group such as to maximize the expected profit:

$$\text{maximize} \sum_{\mathbf{n},\mathbf{f},\mathbf{w}} \mathrm{PROFIT}_{\mathbf{n},\mathbf{f},\mathbf{w}} c_{\mathbf{n},\mathbf{f},\mathbf{w}}. \qquad (19)$$

5 Computed example

We use the same raw material data as in [6], and the distribution of the 43,949 carcasses is seen in Figure 5. Only few carcasses with a slaughter weight above 99 kg exist which is due to a price penalty imposed onto heavy pigs. The model is used to define sorting groups on a grid spanned by 20 cells in the fat layer dimension and 37 cells in the weight dimension. This is equivalent to the situation in [6] and the boundaries of the domain are defined by the parameters:

$$
\begin{aligned}
\bar{f}_{\min} &= 7.5\text{mm}, \quad \bar{f}_{\max} = 26.5\text{mm}, \quad \Delta_f = 1 \\
\bar{w}_{\min} &= 63.5\text{kg}, \quad \bar{w}_{\max} = 99.5\text{kg}, \quad \Delta_w = 1.
\end{aligned}
$$

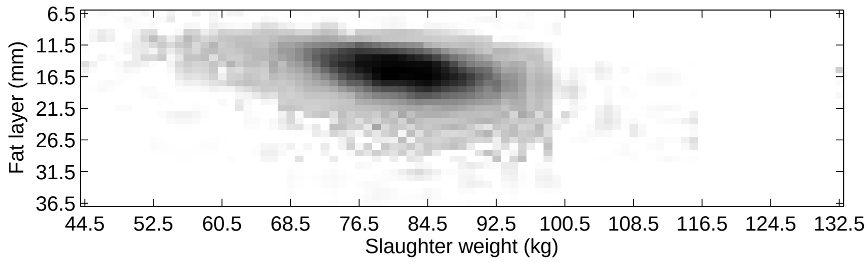


Figure 5: Distribution of carcasses over the extended domain. The colors indicate the amount of carcasses in each cell.

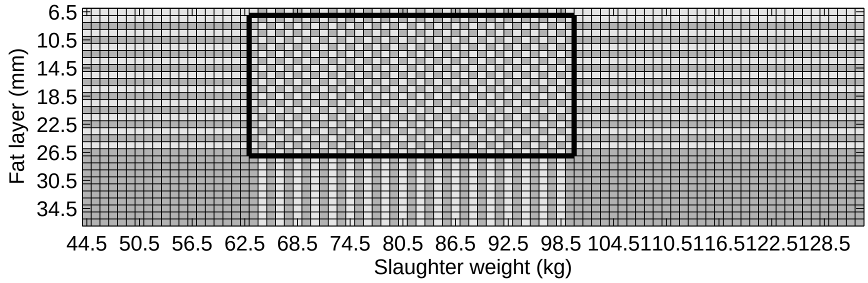


Figure 6: Illustration of sorting groups. The model is allowed to select the product mix individually for cells in the central domain. In the extended domain, the product mixes are identical to the nearest cell on the boundary of the central domain.

We observe from Figure 5 that some of the 43,949 carcasses fall outside the above domain. In [6], profit and product yields were calculated for each individual carcass, and the grid was used only to illustrate the sorting groups. The carcasses falling outside the domain were placed in the sorting groups at the boundary of the domain. The current model is based on profit and product yield computations performed directly on the discrete grid. This may result in significant errors if carcasses falling far outside the domain are represented in the cells at the boundary. The domain is therefore extended to cover all carcasses such that:

$$f_{\min} = 6.5\text{mm}, \quad f_{\max} = 36.5\text{mm}, \quad \Delta_f = 1$$
$$w_{\min} = 44.5\text{kg}, \quad w_{\max} = 132.5\text{kg}, \quad \Delta_w = 1,$$

define an extended domain with $F = 31$ cells in the fat layer dimension and $W = 89$ cells in the weight dimension. Furthermore, we restrict the sorting groups of the cells which fall outside the central domain to be equal to the sorting group at the boundary of the central domain using the parameters $\text{FIX}_{\mathbf{f},\mathbf{w}}$. In this way, carcasses falling outside the central domain will be used for the same product mix as the carcasses falling at the boundary of the central domain. The situation is illustrated in Figure 6.

Finally, we use a discrete blurring operator as defined in Appendix B, as-

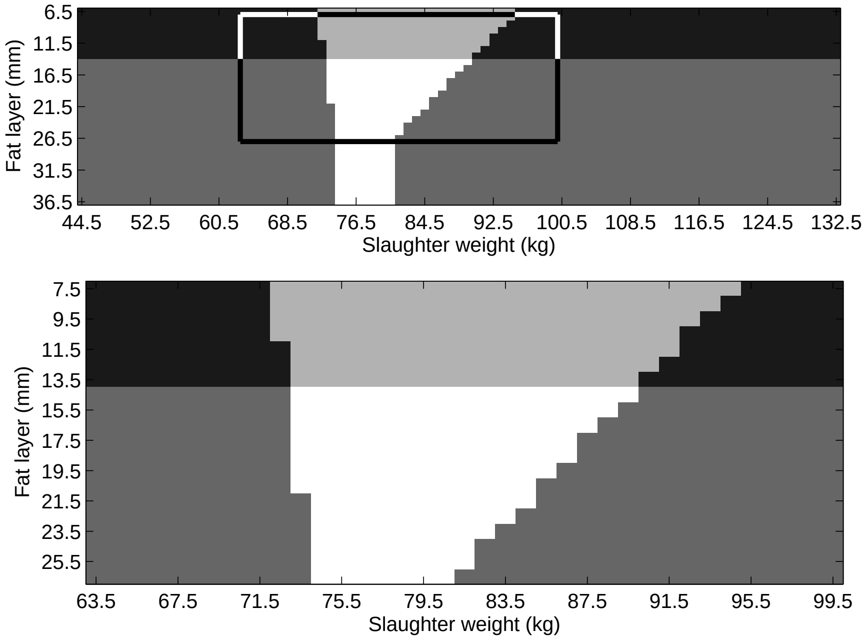


Figure 7: Theoretically optimal production of product mix M_1 (black), M_2 (dark gray), M_3 (light gray), and M_4 (white). Top: Extended domain. Bottom: central domain enlarged.

suming a uniform distribution of carcasses within each cell and an upsampling factor of $a = 32$.

Example 1a

We consider the perfect situation with no measurement errors, i.e. $\sigma_f = \sigma_w = 0$, such that each cell is optimally used for one product mix regardless of the raw material distribution. The optimal sorting groups are computed based on an even distribution of pigs over the central domain in order to avoid cells with no carcasses. We allow each of the $20 \times 37 = 740$ cells in the central domain to be used for any of the four product mixes. Figure 7 shows how the carcasses from each cell should be used in order to obtain the theoretically maximum profit. The figure also illustrates how the sorting groups are extended outside the central domain; especially visible for the white sorting group which is extended downwards. Finally, we fix the sorting groups and apply the true raw material distribution from Figure 5. The expected profit based on the 43,949 carcasses is calculated and seen in Table 3 as case 1.

Example 1b

We now set $\sigma_f = 1.28$, which reflects the actual error of the fat layer measurements at the slaughterhouses. The sorting groups are fixed from Example 1a, but the expected fraction of carcasses in each cell is now based on the distribution of measurement errors. The expected profit is reported in Table 3 as case 2, and is seen to be lower than for case 1 because of the expected misplacement into non-optimal sorting groups. Figure 8 illustrates this misplacement within the central domain. The top-most image shows the probability that product mix M_4 is produced given the sorting groups in Figure 7. We note that the probability is blurred across the sorting group boundary due to the assumed distribution of measurement errors. The estimated number of carcasses used for product mix M_4 is illustrated in the bottom image. A considerable number of carcasses with a true fat layer below 14 mm are assumed to be used for product mix M_4 even though the optimal use of these carcasses would be product mix M_3.

Example 1c

We now compute the optimal sorting groups while explicitly taking into account the distribution of measurement errors. This results in the sorting groups for the central domain shown in Figure 9. The sorting groups are similar, but *not* identical to the sorting groups in Figure 7. For instance, it is no more optimal to put carcasses with a measured fat layer of 12 to 14 mm and a weight above 90 kg into a sorting group for product mix M_1. The reason is that for these cells a larger profit loss is obtained by erroneously producing M_1 if M_2 is optimal than visa versa. The most heavy pigs are even now optimally used for product mix M_4 due to presence of heavier carcasses outside the central domain. The expected profit is shown as case 3 in Table 3. Based on the total supply of $\mathrm{R} = 43,949$ carcasses, we expect to gain DKK 60,538 as compared to case 2, i.e. by changing the sorting groups from the ones shown in Figure 7 to the ones in Figure 9. The difference for the Danish slaughterhouses on an annual basis would be DKK 27.6 mio assuming an annual production of 20 mio pigs.

The sorting groups in Figure 9 can be compared to the the images reported in [6] where the optimal raw material uses are calculated explicitly based on simulation of measured parameters. Indeed, the optimal raw material uses are similar, which indicates that the proposed model correctly accounts for the measurement errors.

Simulation study

We address the discretization error in representing the raw material distribution on a discrete grid. In the above example, all profit calculations were based on the center of the cells. However, the profit and product yield change within the cells such that the estimated profit depends on the actual placement of

	Case 1	Case 2	Case 3
Profit in DKK	38,256,455	37,993,594	38,054,132

Table 3: Optimal expected profits assuming no measurement errors (case 1), the expected profit using the same sorting groups but sort according to measured parameters (case 2), and the optimal expected profit when measurement errors are taken into account when computing the optimal sorting groups (case 3).

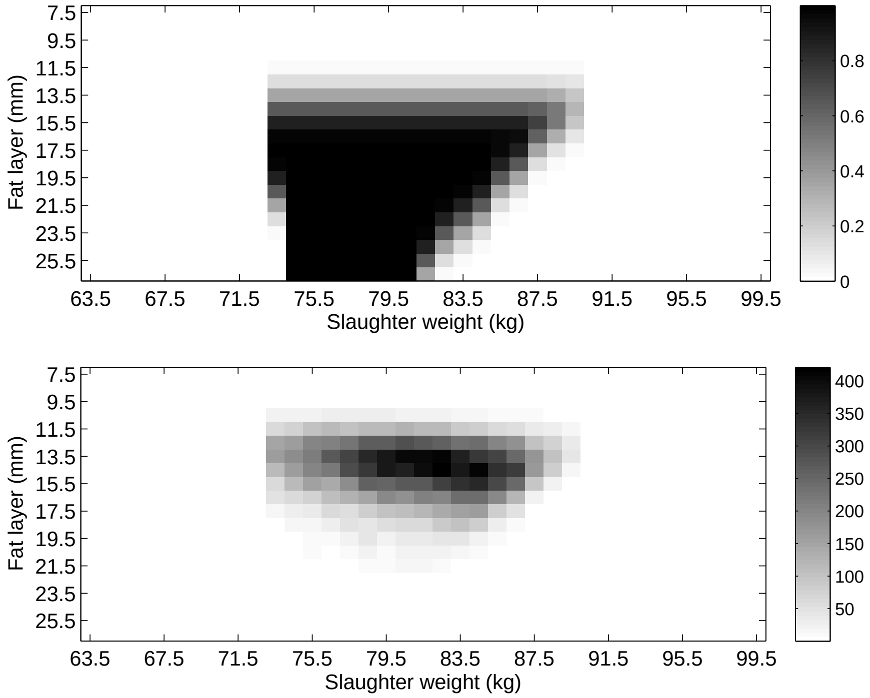


Figure 8: Top: probability for producing product mix M_4 when the sorting categories in Figure 7 are used, and the fat layer measurement errors have $\sigma_f = 1.28$. Bottom: actual number of pigs (of 43,949 in total) used for product mix M_4.

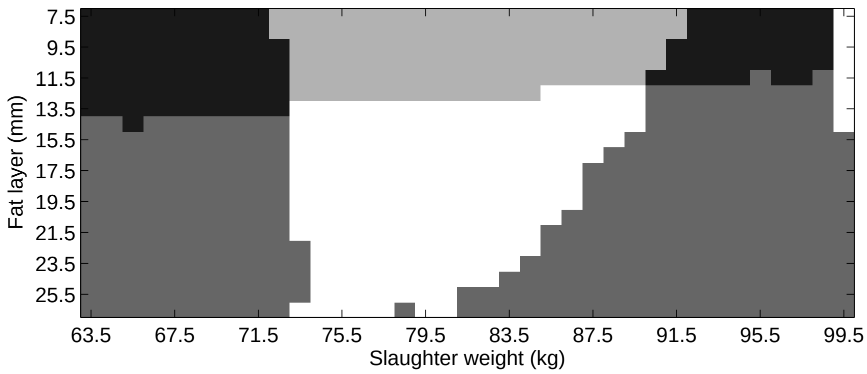


Figure 9: Optimal sorting groups while taking into account that values of the fat layer are contaminated by measurement errors with $\sigma_f = 1.28$. The gray scale levels are equivalent to Figure 7.

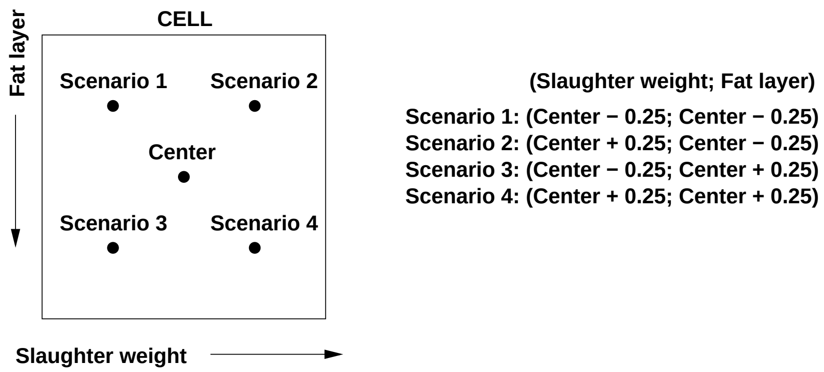


Figure 10: Different assumptions on the carcass placement within the cells.

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Case 1	38,008,982	38,221,215	37,765,183	37,972,323
Case 2	38,077,391	38,285,412	37,824,149	38,035,561
Difference	68,409	64,197	58,966	63,238

Table 4: Expected profit in DKK for each of the four scenarios using the default sorting groups from Figure 7 (Case 1), and the computed optimal sorting groups (Case 2), respectively.

the raw materials within the cells. We consider four scenarios in which the profit calculations are based on different points within the cells; see Figure 10. For instance, the carcasses are assumed to be lighter and skinnier in scenario 1 as compared to the centered case, and heavier and fatter in scenario 4, etc. Table 4 shows the estimated profit obtained using the default sorting groups from Figure 7 and the optimal sorting groups computed by the model. The optimal sorting groups have a structure very similar to the sorting groups shown in Figure 9 for the centered case. The results show that the placement of the carcasses within the cells severely impact the calculated profit. Heavier carcasses (scenario 2 and 4) result in higher profit than lighter carcasses (scenario 1 and 3), and lean carcasses (scenario 1 and 2) result in higher profit than more fat carcasses (scenario 3 and 4). However, the difference from using default sorting groups to using optimal sorting groups is approximately DKK 60,000 to 70,000 in all four scenarios as was the case in Example 1c.

Finally, we perform a simulation study where each of the 43,949 carcasses are processed individually and normally distributed noise with zero mean and standard deviation $\sigma_f = 1.28$ is added to the fat layer parameters in order to simulate the measurement noise. Each carcass is then placed in the cell corresponding to the slaughter weight and the simulated measured fat layer and used for the product mix as described by each of the sorting groups. The profit calculations are based on the assumed true slaughter weights and fat layers as given for the individual carcasses. Table 5 shows the minimum, maximum and mean of the obtained profit over 10 realizations of the measured fat layers. Obviously, the carcasses are not necessarily centered in the cells, and the mean

Sorting groups	Profit Min	Max	Mean
Figure 7	37,962,870	37,973,447	37,969,059
Figure 9	38,025,022	38,031,269	38,027,333
Scenario 1	38,021,385	38,027,756	38,023,490
Scenario 2	38,026,262	38,030,903	38,027,626
Scenario 3	38,024,917	38,031,207	38,027,287
Scenario 4	38,024,291	38,030,338	38,026,462

Table 5: Simulated profit when sorting the true carcasses according to various sorting groups based on 10 realizations of the fat layer measurements.

profits are seen to be lower than the estimated profit reported in Example 1c. However, the profits are well within the span of profits reported in the four scenarios above. Moreover, a profit gain of more than DKK 54,000 as compared to the default sorting groups in Figure 7 is obtained using the sorting groups in Figure 9 as well of any of the similar sorting groups underlying scenarios 1 through 4.

6 Discussion and conclusion

The model illustrates how measurement errors can explicitly be taken into account when designing theoretically optimal sorting groups for pig carcasses for production of a number of product alternatives. Moreover, the example shows how the theoretically optimal sorting strategies change when measurement errors occur.

The modeling capabilities are interesting for a number of reasons. Obviously, the raw material distribution has an impact on the optimal sorting categories when measurement errors are considered because the number of misplaced pig carcasses is influenced by the raw material distribution; see e.g. Eq. (17). This means that the same sorting strategy is not necessarily optimal for two different raw material distributions. In [5], it was studied how to plan the supply of livestock in order to maintain a steady production at the slaughterhouse. However, it is relevant to take into account not only the flow of raw materials, but also quality parameters – here represented by slaughter weight and fat layer. It is therefore interesting to use the model on different raw material distributions, e.g. raw materials from different days or seasons, in order to analyze how the optimal sorting is affected by variations in the raw material supply. This approach may indicate that more advanced sorting groups should be considered in order to handle the variation. In a more holistic setting, it is also relevant to consider joint modeling of the supply of live pigs and the sorting groups.

The model should also be extended to include demand requirements; e.g such in order to compute how some of the sorting groups must be expanded in order to support minimum demand requirements. How to change the sorting groups obviously depends on the raw material distribution and the expected misplacement into wrong sorting groups. Models based on the one presented here may be used to compute optimal sorting groups such that the sorting in the cold storage room reflects the expected raw material supply as well as the expected product demand.

It is highlighted that the approach taken in the current paper is to compute the theoretically optimal sorting groups based on the risk for misplacement. Apart from the simulation study, we do not consider actual realizations of the measurement errors. In line with above comments this suggests that additional sensitivity analyses and improved knowledge on raw material supply and demand patterns are important steps toward applying the models in practice. Furthermore, ongoing research deals with more product alternatives and more final products, as well as emerging possibilities to perform additional sorting before the final production.

The paper addresses the sorting groups within the slaughterhouse, which is only part of a larger supply chain as shown in Figure 1. The perspectives for modeling the interaction between several actors is therefore interesting. Moreover, quality and safety aspects are affected by actors throughout the chain, and optimal decisions in one part of the chain will affect the possibilities for profit optimization in other parts of the chain. Future research directions include joint modeling of several aspects such as supply and demand, as well as transportation, primary production and optimal raw material utilization.

Acknowledgements

The research was partly funded by the Danish Food Industry Agency through the project OPUS+ hosted by the Danish Meat Reserch Institute, a division of the Danish Technological Institute.

Appendix A Product yield model

A product yield model is constructed to estimate the value of a pig carcass with given slaughter weight and fat layer when used for a certain product mix. The model is based on the parameters in Table 6. Note that the last product is an auxiliary product only used for the internal calculations. The value of this product is therefore zero. In general, the product weight W_j of each product j if produced from raw materials with fat layer f and slaughter weight w is given as:

$$W_j(f,w) = \mathrm{PW}_j + \mathrm{PF}_j f + \mathrm{PWW}_j w.$$

However, the weight of some products are calculated separately:

$$\begin{aligned} W_{\text{Sundry 3}}(f,w) = {} & W_{\text{Backs w bones}}(f,w) + W_{\text{Breast 1}}(f,w) \\ & + W_{\text{CutOff 2}}(f,w) + W_{\text{Sundry2}}(f,w) \\ & - W_{\text{Backs boneless}}(f,w) - W_{\text{Breast 2}}(f,w) \\ & - W_{\text{CutOff 3}}(f,w) \end{aligned}$$

$$W_{\text{Sundry 4}}(f,w) = W_{\text{Back-end aux}}(f,w) - W_{\text{Ham}}(f,w)$$

$$\begin{aligned} W_{\text{Sundry 5}}(f,w) = {} & W_{\text{Back-end aux}}(f,w) - W_{\text{Ham boneless}}(f,w) \\ & - W_{\text{CutOff 5}}(f,w) \end{aligned}$$

$$
\begin{aligned}
W_{\text{Head}}(f,w) = w &- W_{\text{Shoulder}}(f,w) - W_{\text{Neck}}(f,w) \\
&- W_{\text{Backs w bones}}(f,w) - W_{\text{Breast 1}}(f,w) \\
&- W_{\text{Ham boneless}}(f,w) - W_{\text{CutOff 1}}(f,w) \\
&- W_{\text{CutOff 2}}(f,w) - W_{\text{CutOff 5}}(f,w) \\
&- W_{\text{Sundry 1}}(f,w) - W_{\text{Sundry 2}}(f,w) \\
&- W_{\text{Sundry 5}}(f,w) - W_{\text{Tenderloin}}(f,w)
\end{aligned}
$$

Moreover, if $W_{\text{Sundry 4}}(f,w) < 0.1$ then the weight of Ham and Sundry 4 are corrected such that:

$$
\begin{aligned}
W_{\text{Ham}}(f,w) &= W_{\text{Ham}}(f,w) - (0.1 - W_{\text{Sundry 4}}(f,w)) \\
&\text{and} \\
W_{\text{Sundry 4}}(f,w) &= 0.1
\end{aligned}
\tag{20}
$$

Product (j)	P	PC	PW	PF	PWW	
Shoulder	12	0.00	0.00000	-0.06938	0.10726	1,2,3,4
Neck	13	0.00	0.00000	-0.04096	0.07282	1,2,3,4
Backs w bones	18	-0.20	10.77058	-0.01662	0.01354	1,2
Breast 1	13	-0.20	2.00642	0.04284	0.06002	1,2
Backs boneless	25	-0.20	0.46036	-0.08124	0.08666	3,4
Breast 2	17	-0.20	2.00642	0.04284	0.06002	3,4
Ham	15	-0.20	0.00000	-0.10204	0.27632	1,3
Ham boneless	18	-0.20	-1.11490	-0.19054	0.22874	2,4
CutOff 1	9	-0.10	0.00000	-0.00596	0.00834	1,2,3,4
CutOff 2	9	-0.10	0.00000	-0.00596	0.00834	1,2
CutOff 3	9	-0.10	0.00000	-0.00596	0.00834	3,4
CutOff 5	9	-0.10	0.00000	-0.00596	0.00834	2,4
Sundry 1	3	0.00	-1.95414	0.07922	0.13368	1,2,3,4
Sundry 2	3	0.00	-14.54192	0.11178	0.24410	1,2
Sundry 3	3	0.00	0.00000	0.00000	0.00000	3,4
Sundry 4	3	0.00	0.00000	0.00000	0.00000	1,3
Sundry 5	3	0.00	0.00000	0.00000	0.00000	2,4
Tenderloin	30	0.00	1.20000	0.00000	0.00000	1,2,3,4
Head	3	0.00	0.00000	0.00000	0.00000	1,2,3,4
Back-end aux	0	0.00	-1.58570	-0.10160	0.29790	

Table 6: Parameters for product yield model: P (price), PC (price coefficient), PW (product weight constant), PF (weight fat dependent parameter), PWW (weight weight dependent parameter) – all in DKK. The last column indicates the product mixes that the given product is a part of.

A number of quality deductions QD apply:

$$QD_{\text{Backs w bones}}(f,w) = \begin{cases} 2 & \text{if } W_{\text{Backs w bones}}(f,w) > 7 \\ 0 & \text{otherwise} \end{cases}$$

$$QD_{\text{Backs boneless}}(f,w) = \begin{cases} 2 & \text{if } W_{\text{Backs boneless}}(f,w) > 7 \\ 0 & \text{otherwise} \end{cases}$$

$$QD_{\text{Breast 2}}(f,w) = \begin{cases} 6 & \text{if } W_{\text{Breast 2}}(f,w) > 8 \\ 0 & \text{otherwise} \end{cases}$$

$$QD_{\text{Ham}}(f,w) = \begin{cases} 4 & \text{if } f > 14 \\ 0 & \text{otherwise} \end{cases}$$

Quality deductions are zero for all other products.

Finally, the value of using a pig with parameters f and w for a given product mix is given as:

$$\mathcal{P}_{M_n}(f,w) = \sum_{J_{M_n}} (\mathrm{P}_j - QD_j(f,w) + \mathrm{PC}_j(f - 15.9)) * W_j(f,w),$$

where J_{M_n} is the set of products that are part of the product mix M_n as given in the last column of Table 6.

Appendix B The discrete blurring function

The two-dimensional Gaussian function $\mathcal{G}(f,w)$ from Eq. (6) can be sampled over the bandwidth B in both dimensions such that:

$$\mathrm{G}_{\mathbf{b}_f,\mathbf{b}_w} = \frac{1}{2\pi\sigma_f\sigma_w} e^{-\left(\frac{(\mathbf{b}_f - m)^2}{2\sigma_f^2} + \frac{(\mathbf{b}_w - m)^2}{2\sigma_w^2}\right)}, \tag{21}$$

can be used as a discrete approximation of the blurring function in the optimization model. All variables are defined in Section 4. However, discretization errors occur and all carcasses are assumed to lie in the center of the cells. The latter means that small measurement errors may not result in any misplacement, whereas even small measurement errors in practice will lead to some misplacement of carcasses. A more dense discretization will reduce the discretization errors, but the size of the MIP model will increase. We therefore define a discrete blurring operator based on an upsampled grid and assume, for the construction of the blurring operator, that the carcasses are uniformly distributed over the cells. For simplicity, we derive the equations only for the fat layer dimension and leave out the weight indices. Similar calculations can be performed for the slaughter weights and applied independently.

We define the upsampling factor $a = 2p$, $p \in \mathbb{N}$ such that $\widetilde{\mathcal{F}} = \{\ldots, \frac{1}{2}, \frac{1}{2} + \frac{1}{a}, \ldots, F + \frac{1}{2} - \frac{1}{a}, \ldots\}$ is the set of points defining a grid. The original index $\mathbf{f}_i$ is subsampled onto $[\mathbf{f}_i - \frac{1}{2}, \mathbf{f}_i - \frac{1}{2} + \frac{1}{a}, \ldots, \mathbf{f}_i + \frac{1}{2} - \frac{1}{a}]$. We consider a Gaussian function sampled over the upsampled fat layer indices $\tilde{\mathbf{f}} \in \widetilde{\mathcal{F}}$ such that:

$$G(\tilde{\mathbf{f}}) = \frac{1}{a} \frac{1}{\sigma_f\sqrt{2\pi}} e^{-\frac{(\tilde{\mathbf{f}} - f_m)^2}{2\sigma_f^2}}, \tag{22}$$

where $\frac{1}{a}$ is the grid spacing. We also define an upsampled sorting map $\tilde{s}_{\mathbf{n}}(\tilde{\mathbf{f}})$ which equals the original binary sorting map $s_{\mathbf{n},\mathbf{f}}$ for all coinciding indices:

$$\tilde{s}_{\mathbf{n}}(\tilde{\mathbf{f}}) = \begin{cases} s_{\mathbf{n},\mathbf{f}} & \text{for} \quad \tilde{\mathbf{f}} \in [f_{\min},\, f_{\min}+1,\, \ldots,\, f_{\max}] \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

Finally, we assume a uniform distribution of carcasses within each cell:

$$U(\tilde{\mathbf{f}}) = \begin{cases} \frac{1}{a} & \text{for} \quad \tilde{\mathbf{f}} \in [-\frac{1}{2};\, \frac{1}{2} - \frac{1}{a}] \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

The uniform distribution and the measurement errors can be applied to the sorting groups by two convolutions such that:

$$P_{\mathbf{n}}(\tilde{\mathbf{f}}) = G(\tilde{\mathbf{f}}) * \Big(U(\tilde{\mathbf{f}}) * \tilde{s}_{\mathbf{n}}(\tilde{\mathbf{f}}) \Big) \tag{25}$$

is the probability that a carcass is used for product mix M_n given on the upsampled grid. Convolution is associative such that

$$\begin{aligned} P_{\mathbf{n}}(\tilde{\mathbf{f}}) &= \Big(G(\tilde{\mathbf{f}}) * U(\tilde{\mathbf{f}}) \Big) * \tilde{s}_{\mathbf{n}}(\tilde{\mathbf{f}}) & (26) \\ &= \left(\sum_{\bar{\mathbf{f}} \in \tilde{\mathcal{F}}} G(\tilde{\mathbf{f}} - \bar{\mathbf{f}}) U(\bar{\mathbf{f}}) \right) * \tilde{s}_{\mathbf{n}}(\tilde{\mathbf{f}}). & (27) \end{aligned}$$

The summation limits can be reduced as $U(\tilde{\mathbf{f}}) = \frac{1}{a}$ for $\tilde{\mathbf{f}} \in [-\frac{1}{2};\, \frac{1}{2} - \frac{1}{a}]$ and zero otherwise, such that:

$$P_{\mathbf{n}}(\tilde{\mathbf{f}}) = \left(\frac{1}{a} \sum_{\bar{\mathbf{f}}=-\frac{1}{2}}^{\frac{1}{2}-\frac{1}{a}} G(\tilde{\mathbf{f}} - \bar{\mathbf{f}}) \right) * \tilde{s}_{\mathbf{n}}(\tilde{\mathbf{f}}). \tag{28}$$

The next convolution results in:

$$\begin{aligned} P_{\mathbf{n}}(\tilde{\mathbf{f}}) &= \frac{1}{a} \sum_{\hat{\mathbf{f}} \in \tilde{\mathcal{F}}} \left(\sum_{\bar{\mathbf{f}}=-\frac{1}{2}}^{\frac{1}{2}-\frac{1}{a}} G(\tilde{\mathbf{f}} - \hat{\mathbf{f}} - \bar{\mathbf{f}}) \right) \tilde{s}_{\mathbf{n}}(\hat{\mathbf{f}}) & (29) \\ &= \frac{1}{a} \sum_{\hat{\mathbf{f}}=1}^{F} \left(\sum_{\bar{\mathbf{f}}=-\frac{1}{2}}^{\frac{1}{2}-\frac{1}{a}} G(\tilde{\mathbf{f}} - \hat{\mathbf{f}} - \bar{\mathbf{f}}) \right) s_{\mathbf{n},\hat{\mathbf{f}}}, & (30) \end{aligned}$$

where the upsampled indices of the first sum can be exchanged by the original indices because $\tilde{s}_{\mathbf{n}}(\tilde{\mathbf{f}})$ is zero for all indices different from the original. Finally, we sum up the probabilities on the upsampled grid to get the probabilities on the original grid:

$$
\begin{aligned}
P_{\mathbf{n},\mathbf{f}} &= \sum_{\tilde{\mathbf{f}}=\mathbf{f}-\frac{1}{2}}^{\mathbf{f}+\frac{1}{2}-\frac{1}{a}} P_{\mathbf{n}}(\tilde{\mathbf{f}}) && (31)\\
&= \sum_{\tilde{\mathbf{f}}=\mathbf{f}-\frac{1}{2}}^{\mathbf{f}+\frac{1}{2}-\frac{1}{a}} \frac{1}{a} \sum_{\hat{\mathbf{f}}=1}^{F} \left(\sum_{\bar{\mathbf{f}}=-\frac{1}{2}}^{\frac{1}{2}-\frac{1}{a}} G(\tilde{\mathbf{f}}-\hat{\mathbf{f}}-\bar{\mathbf{f}}) \right) s_{\mathbf{n},\hat{\mathbf{f}}} && (32)\\
&= \sum_{\hat{\mathbf{f}}=1}^{F} \left(\frac{1}{a} \sum_{\tilde{\mathbf{f}}=-\frac{1}{2}}^{\frac{1}{2}-\frac{1}{a}} \sum_{\bar{\mathbf{f}}=-\frac{1}{2}}^{\frac{1}{2}-\frac{1}{a}} G(\tilde{\mathbf{f}}-\hat{\mathbf{f}}+\mathbf{f}-\bar{\mathbf{f}}) \right) s_{\mathbf{n},\hat{\mathbf{f}}} && (33)\\
&= \sum_{\hat{\mathbf{f}}=1}^{F} H(\mathbf{f}-\hat{\mathbf{f}}) s_{\mathbf{n},\hat{\mathbf{f}}}. && (34)
\end{aligned}
$$

We note that this defines a convolution on the original grid with the convolution kernel $H(\mathbf{f})$ given by:

$$
\begin{aligned}
H(\mathbf{f}) &= \frac{1}{a} \sum_{\tilde{\mathbf{f}}=-\frac{1}{2}}^{\frac{1}{2}-\frac{1}{a}} \sum_{\bar{\mathbf{f}}=-\frac{1}{2}}^{\frac{1}{2}-\frac{1}{a}} G(\mathbf{f}+\tilde{\mathbf{f}}-\bar{\mathbf{f}}) && (35)\\
&= \frac{1}{a^2\sigma_f\sqrt{2\pi}} \sum_{\tilde{\mathbf{f}}=-\frac{1}{2}}^{\frac{1}{2}-\frac{1}{a}} \sum_{\bar{\mathbf{f}}=-\frac{1}{2}}^{\frac{1}{2}-\frac{1}{a}} e^{-\frac{\mathbf{f}+\tilde{\mathbf{f}}-\bar{\mathbf{f}}-f_m}{2\sigma_f^2}}. && (36)
\end{aligned}
$$

Finally, a two-dimensional blurring function is computed by multiplying Eq. (36) with a similar function describing the blurring in the weight dimension. By exchanging the primary indices to the bandwidth sets used in the optimization model, we get the discrete blurring operator:

$$
\mathrm{H}_{\mathbf{b}_f,\mathbf{b}_w} = \frac{1}{2\pi a^4 \sigma_f \sigma_w} \sum_{\tilde{\mathbf{f}}=-\frac{1}{2}}^{\frac{1}{2}-\frac{1}{a}} \sum_{\bar{\mathbf{f}}=-\frac{1}{2}}^{\frac{1}{2}-\frac{1}{a}} \sum_{\tilde{\mathbf{w}}=-\frac{1}{2}}^{\frac{1}{2}-\frac{1}{a}} \sum_{\bar{\mathbf{w}}=-\frac{1}{2}}^{\frac{1}{2}-\frac{1}{a}} e^{-\left(\frac{(\mathbf{b}_f+\tilde{\mathbf{f}}-\bar{\mathbf{f}}-m)^2}{2\sigma_f^2}+\frac{(\mathbf{b}_w+\tilde{\mathbf{w}}-\bar{\mathbf{w}}-m)^2}{2\sigma_w^2}\right)}.
$$

References

[1] v.d. Broek, J., Schütz, P., Stougie, L., Tomasgard, A. (2006). Location of slaughterhouses under economies of scale. *European J of Operational Research*, 175, 740–750.

[2] Ding, H., Benyoucef, L., and Xie, X. (2009). Stochastic multi-objective production-distribution network design using simulation-based optimization, *Int J of Production Research*, 47(2), 479–505.

[3] Dupuy, C., Botta-Genoulaz, V., Guinet, A. (2005). Batch dispersion model to optimise traceability in food industry, *J of Food Engineering*, 70, 333–339.

[4] Lütke Entrup, M., Günther H.-O., van Beek, P., Grunow, M., and Seiler, T. (2005). Mixed-Integer Linear Programming approaches to shelf-life-integrated planning and scheduling in yoghurt production, *Int J of Production Research*, 43(23), 5071–5100.

[5] Gribkovskaia, I., Gullberg, B.O., Hovden K.J., and Wallace, S.W. (2006). Optimization model for a livestock collection problem, *Int J of Physical Distribution & Logistics Management,* 36(2), 136–152.

[6] Kjærsgaard, N. (2008). Evaluation of Different Sorting Criteria and Strategies Using Mathematical Programming, Technical Report, DTU Informatics, Technical University of Denmark, March.

[7] Kjærsgaard, N. (2008). Optimization of the Raw Material Use at Danish Slaughterhouses, Ph.D Thesis, IMM-PHD-2008-197, Department of Management Engineering, Technical University of Denmark.

[8] Ohlmann, J.W. and Jones, P.C. (2010). An integer programming model for optimal pork marketing, *Annals of Operations Research,* DOI 10.1007/s10479-008-0466-3.

[9] Oppen, J., Lœkketangen, A. and Desrosiers, J. (2009). Solving a rich vehicle routing and inventory problem using column generation, *Computers and Operations Research,* 37(7), 1208–1317.

[10] Yücel, E., Karaesmen, F., Sibel Salman, F., Türkay M. (2009). Optimizing product assortment under customer-driven demand substitution, *European J of Operational Research,* 199(3), 759–768.

[11] Wang X., Li D. and O'Brian, C. (2009). Optimization of traceability and operations planning: an integrated model for perishable food production, *Int J of Production Research,* 47(11), 2865–2886.

Slaughterhouses are major players in the pork supply chain, and supply and demand must be matched in order to generate the highest profit. In particular, carcasses must be sorted in order to produce the "right" final products from the "right" carcasses. We develop a mixed-integer programming (MIP) model for computing the optimal sorting of carcasses according to two parameters; slaughter weight and fat layer. Moreover, we consider a new approach for dealing with expected measurement errors. The results provide insight into how sorting groups should be designed in order to improve the profit at slaughterhouses. Finally, we comment on the expected effect of variations in the raw material supply and the demand as well as future research concerning joint modelling of supply chain aspects.

978-87-90855-77-2

DTU Management Engineering
Department of Management Engineering
Technical University of Denmark

Produktionstorvet
Building 424
DK-2800 Kongens Lyngby
Denmark
Tel. +45 45 25 48 00
Fax +45 45 93 34 35

www.man.dtu.dk